# Time crystals and synchronization in quantum systems

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# Outline

- Introduction to quantum many-body open systems
- Time crystals in closed and open systems
- Possible applications in quantum sensing and quantum thermodynamics

# Time Crystals

- Do laws of nature allow for the existence of a time-crystalline phase? ✓
- How to define/characterise a time crystal? ✓
- Where to look for it? ✓

V. Khemani, R. Moessner, and S.L. Sondhi, arXiv:1910.10745
K. Sacha and J. Zakrzewski, Rep. Prog. Phys. 81, 016401 (2018)
M. P. Zaletel, M. Lukin, C. Monroe, C. Nayak, and F. Wilczek, Rev. Mod. Phys. 95, 031001 (2023)
K. Sacha, Time Crystals, Springer (2020)

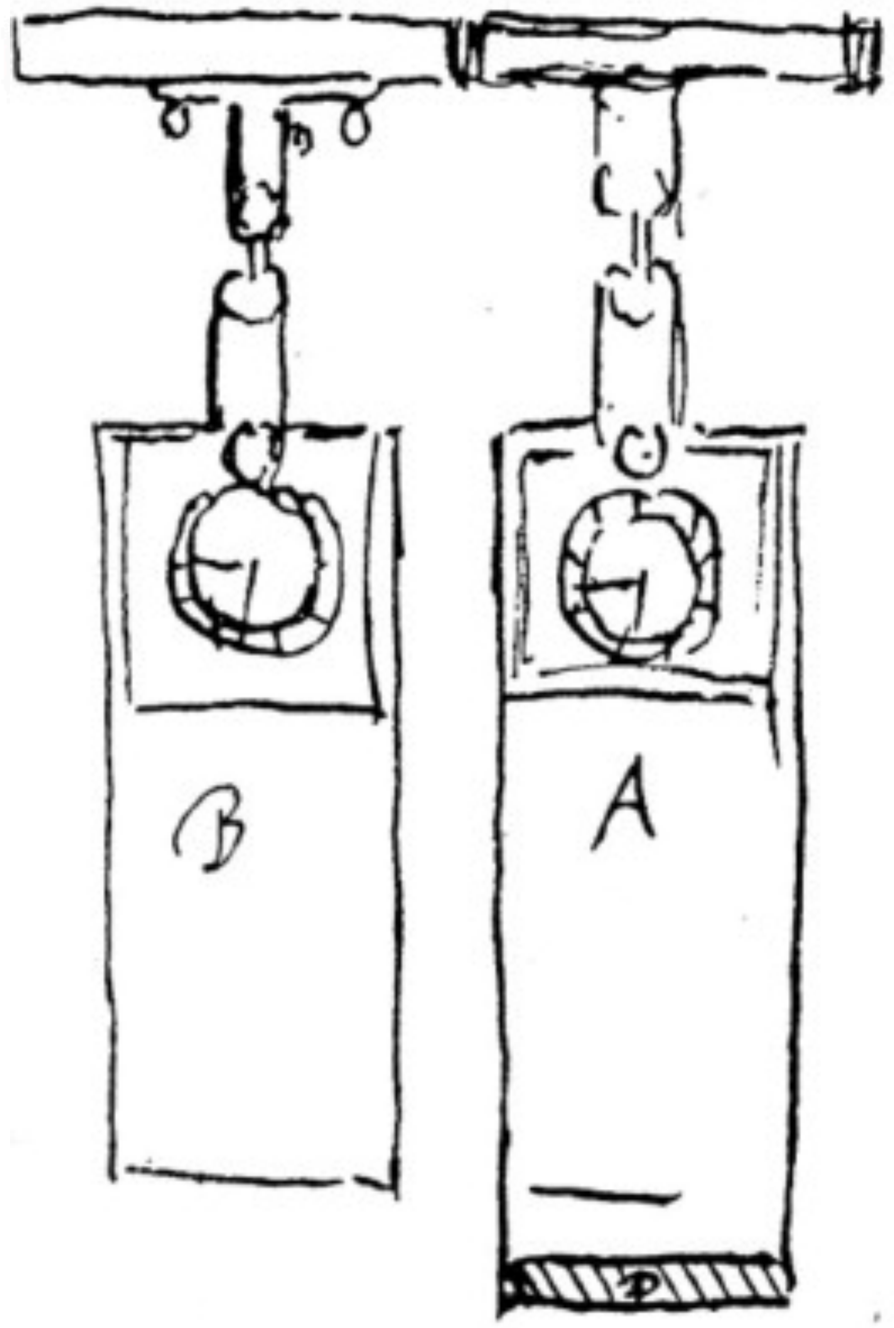
- Is it "useful" for possible applications in quantum technologies?

## “Many-body” limit cycles as time-crystals in open systems

These limit cycles can be understood as a macroscopic synchronized dynamics characterized by a time-dependent order parameter

## 1665.

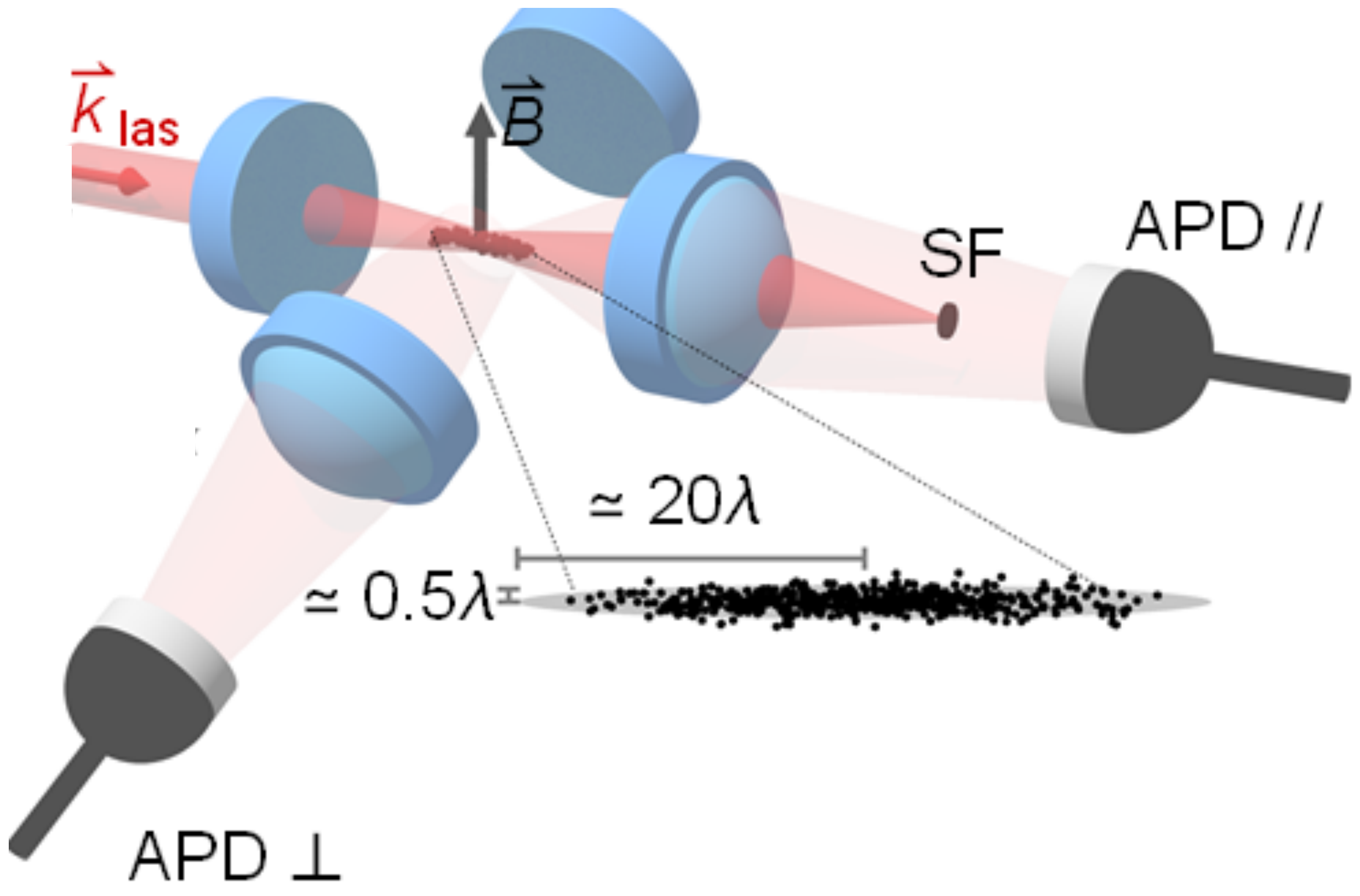
[Fig. 75.][1])



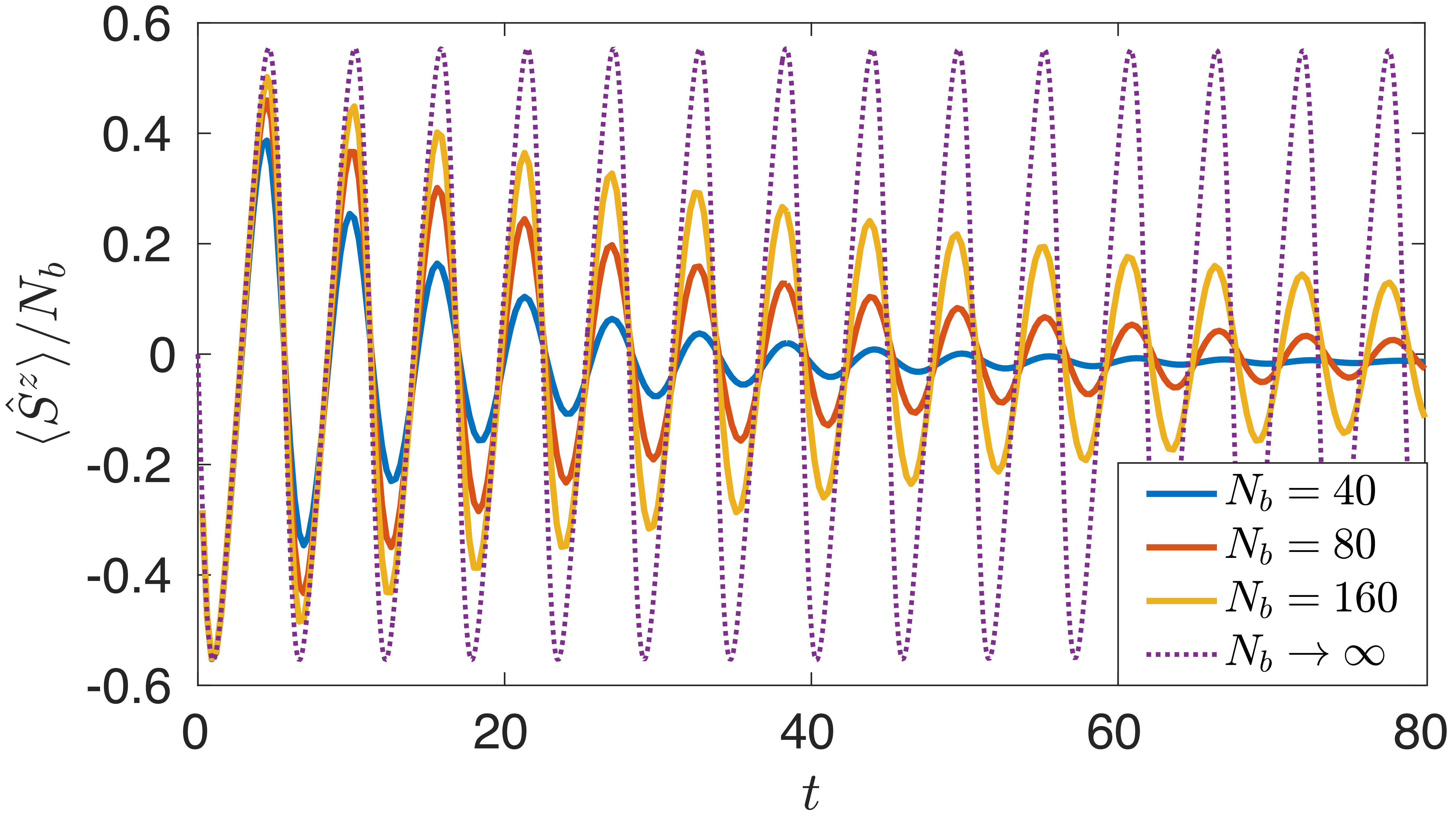

22 febr. 1665.

Diebus 4 aut 5 horologiorum duorum novorum in quibus catenulæ [Fig. 75], miram concordiam obſervaveram, ita ut ne minimo quidem exceſſu alterum ab altero ſuperaretur. ſed conſonarent ſemper reciprocationes utriusque perpendiculi. unde cum parvo ſpatio inter ſe horologia diſtarent, ſympathiæ quandam[3]) quasi alterum ab altero afficeretur ſuſpicari cœpi. ut experimentum caperem turbavi alterius penduli reditus ne ſimul incederent ſed quadrante horæ poſt vel ſemihora rurſus concordare inveni.

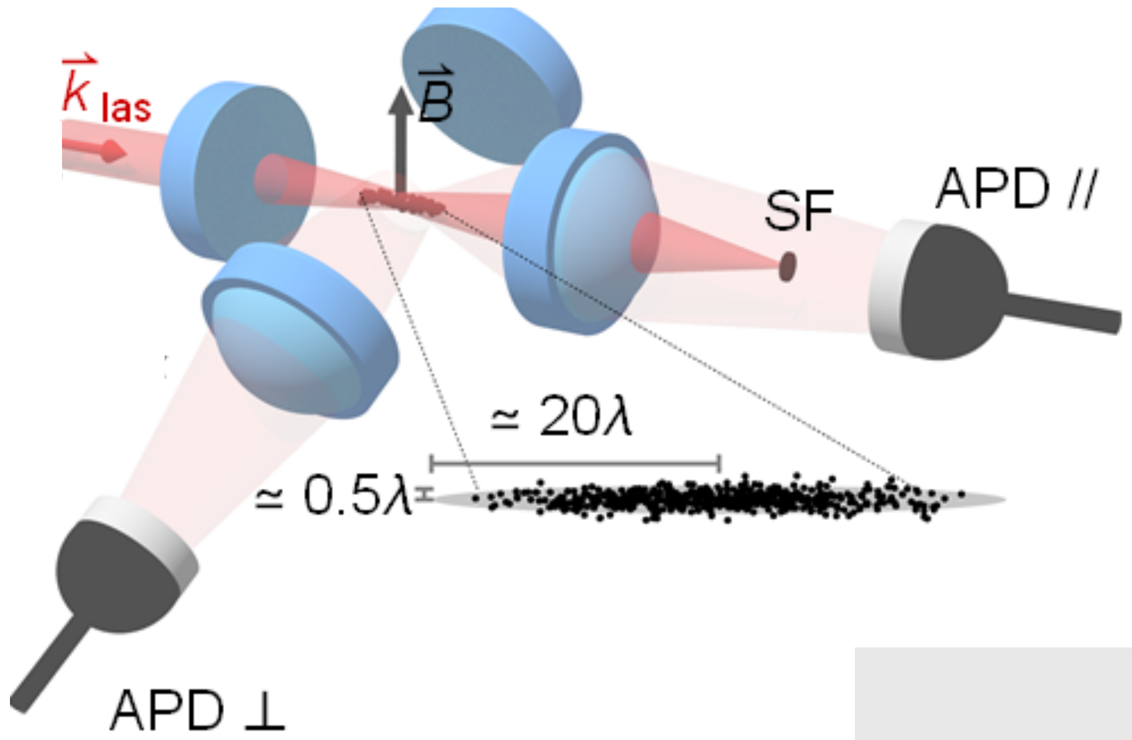
Pendebant horologia ex ſuo quodque tigno 3 circiter pollicum craſſitudine quorum extrema ſedibus duabus[4]) pro fulcris innitebantur. cumque tigna juxta ſe mutuo ſecundum longitudinem jacerent, horologium alterum B non plane a latere erat horologio A ſed antrorſum prominebat. B aliquanto etiam brevius erat quam A neque inferiùs plumbum habebat quod in horologio A notatur D. Singula credo cum ponderibus et plumbo ad faciendum æquilibrium intus poſito ad 80 vel 90 libras pendebant vel A aliquanto amplius ob pondus D[5]). per-



$\vec{k}_{las}$
$\vec{B}$
SF
APD //
≃ 20λ
≃ 0.5λ
APD ⊥

$\langle \hat{S}^z \rangle / N_b$
$t$
$N_b = 40$
$N_b = 80$
$N_b = 160$
$N_b \to \infty$
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
0
20
40
60
80



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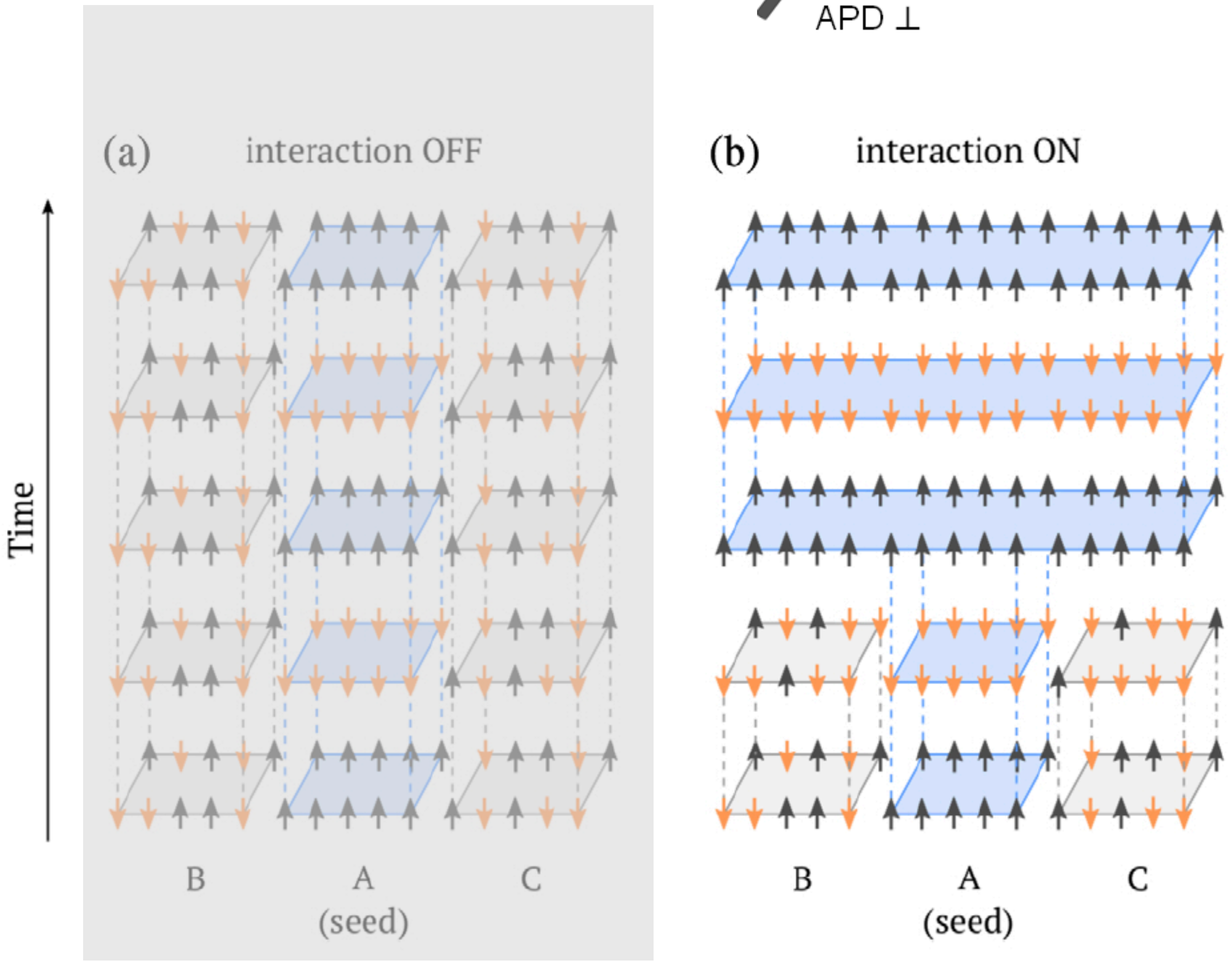
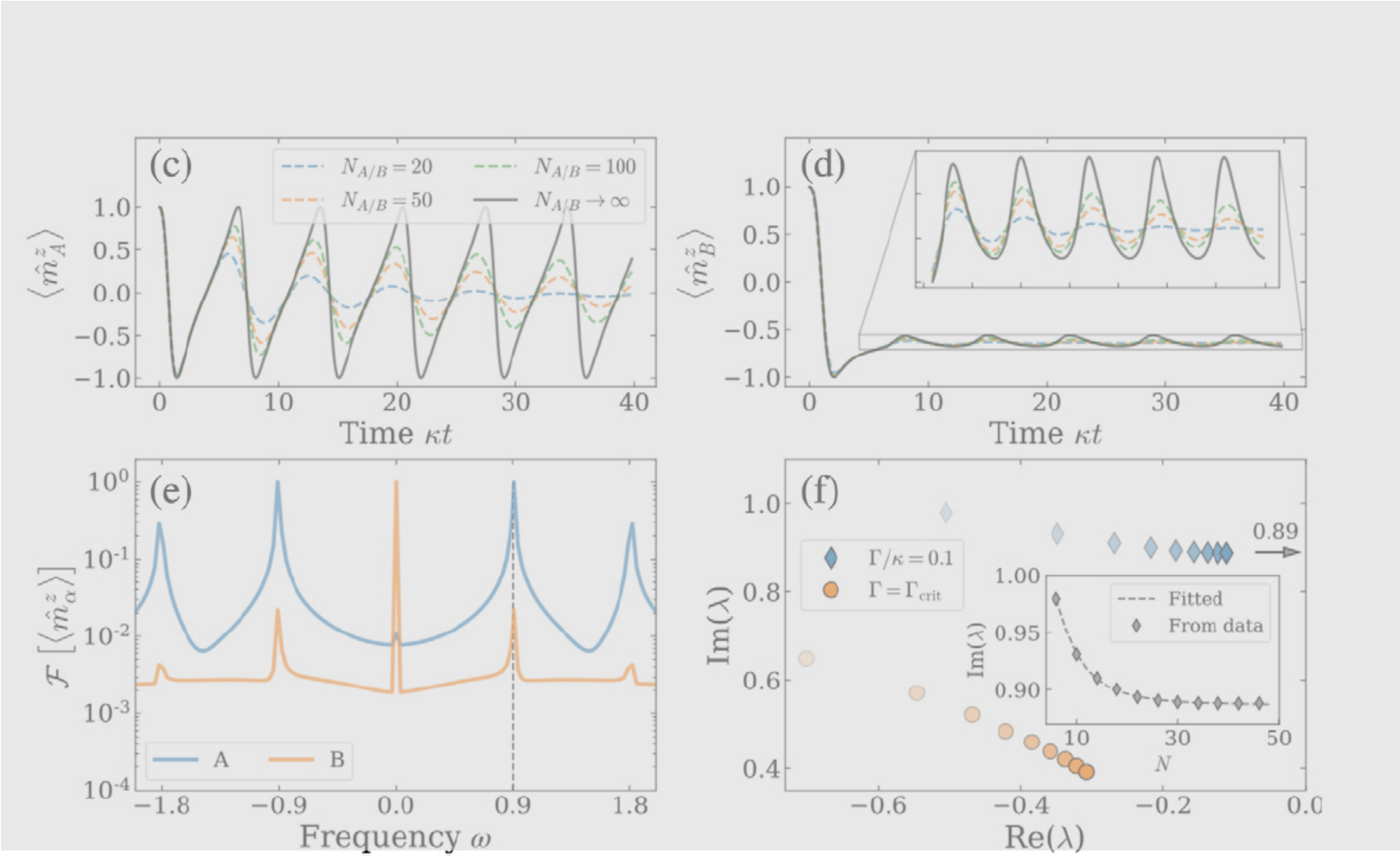


(a) interaction OFF
(b) interaction ON
Time
B
A
(seed)
C
B
A
(seed)
C

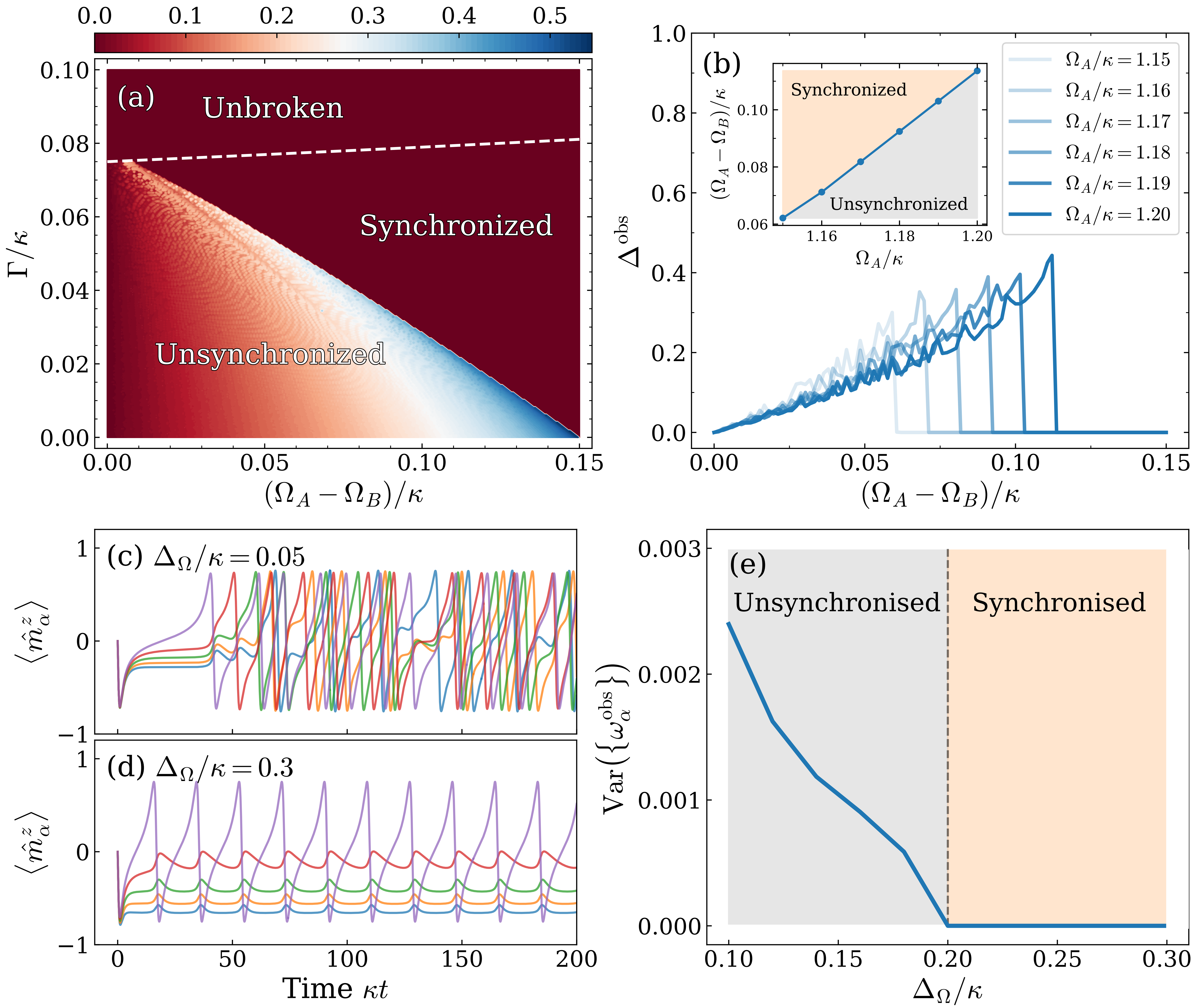



(c)
$N_{A/B} = 20$
$N_{A/B} = 100$
$N_{A/B} = 50$
$N_{A/B} \to \infty$
$\langle \hat{m}_A^z \rangle$
Time $\kappa t$
(d)
$\langle \hat{m}_B^z \rangle$
Time $\kappa t$
(e)
$\mathcal{F}[\langle \hat{m}_\alpha^z \rangle]$
A
B
Frequency $\omega$
(f)
$\Gamma/\kappa = 0.1$
$\Gamma = \Gamma_{crit}$
0.89
Fitted
From data
Im(λ)
N
Re(λ)

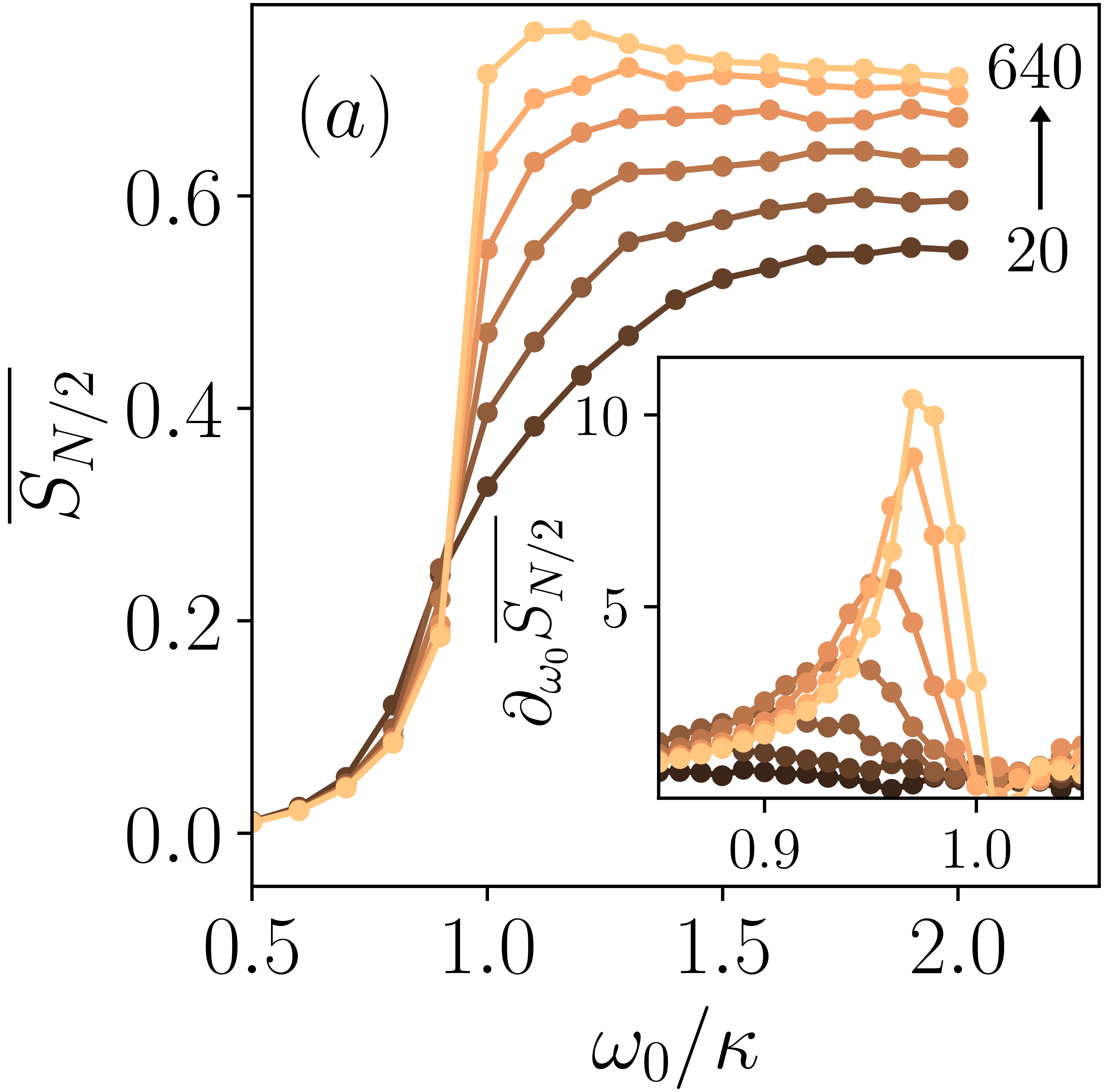
# Synchronization

M. Hajdušek, P. Solanki, R. Fazio, and S. Vinjanampathy, Phys. Rev. Lett. **128**, 080603, (2022)

Dynamics of two coupled time crystals with different periods



# Time crystals & Entanglement




G. Passareli, X. Turkeshi, A. Russomanno, P. Lucignano, M. Schirò, and R.Fazio, PRL **132,** 163401 (2024)

# Quantum Technology applications

## In quantum sensing

S. Choi, N.Y. Yao and M.D. Lukin, arXiv:1801.00042 (2017)

V. Montenegro, M. G. Genoni, A. Bayat, and M. G. A. Paris, arXiv:2301.02103 (2023)

F. Iemini, R. Fazio, and A. Sanpera, arXiv:2306.03927 (2023)

A. Cabot, F. Carollo, and I. Lesanovsky, arXiv:2307.13277 (2023)

L. Viotti, M. Huber, R. Fazio, and G. Manzano, soon on the ArXiv

## As a working fluid in quantum heat engines

F. Carollo, K. Brandner, and I. Lesanovsky, Phys. Rev. Lett. 125, 240602 (2020)

# Lindblad dynamics for a time crystal $T = 0$

$$\hat{J}_\alpha = \sum_{i=1}^{N} \hat{\sigma}_{\alpha,i}$$

- A cloud of N (non-interacting) atoms
- Laser driven $\hat{\mathcal{H}} = \omega_0 \hat{J}_x$
- Collective decay $\hat{J}_-$

$$\dot{\rho} = -i\Omega\left[\hat{J}_x, \rho\right] + \frac{\Gamma_0}{J}\left(\hat{J}_-\rho\hat{J}_+ - \frac{1}{2}\left\{\hat{J}_+\hat{J}_x, \rho\right\}\right)$$

$\langle J_z \rangle \neq 0$ — Normal Phase — 1 — Time-crystal Phase — $\langle J_z \rangle = 0$ — $\Omega/\Gamma_0$

F. Iemini et al, Phys. Rev. Lett. **121**, 035301 (2018)

# Thermodynamics of a time-crystals

$T \neq 0$

$$\dot{\rho} = -i\Omega\left[\hat{J}_x, \rho\right] + \frac{\Gamma_\downarrow}{J}\left(\hat{J}_-\rho\hat{J}_+ - \frac{1}{2}\left\{\hat{J}_+\hat{J}_-, \rho\right\}\right) + \frac{\Gamma_\uparrow}{J}\left(\hat{J}_+\rho\hat{J}_- - \frac{1}{2}\left\{\hat{J}_-\hat{J}_+, \rho\right\}\right)$$

$\Gamma_\downarrow = \Gamma_0[n_B(T) + 1]/J$

$\Gamma_\uparrow = \Gamma_0 n_B(T)/J$

# Quantum trajectories

Lindblad dynamics $$\rho(t+\delta t) = \sum_{\alpha} K_\alpha \rho(t) K_\alpha^\dagger$$

$$K_0 = \hat{\mathbb{I}} - \delta t \left[ i\mathcal{H} + \frac{1}{2} \sum_{\alpha \neq 0} L_\alpha^\dagger L_\alpha \right] \qquad K_\alpha = \sqrt{\delta t} L_\alpha \quad \alpha \neq 0$$

Kraus operators $\hat{K}_\alpha$

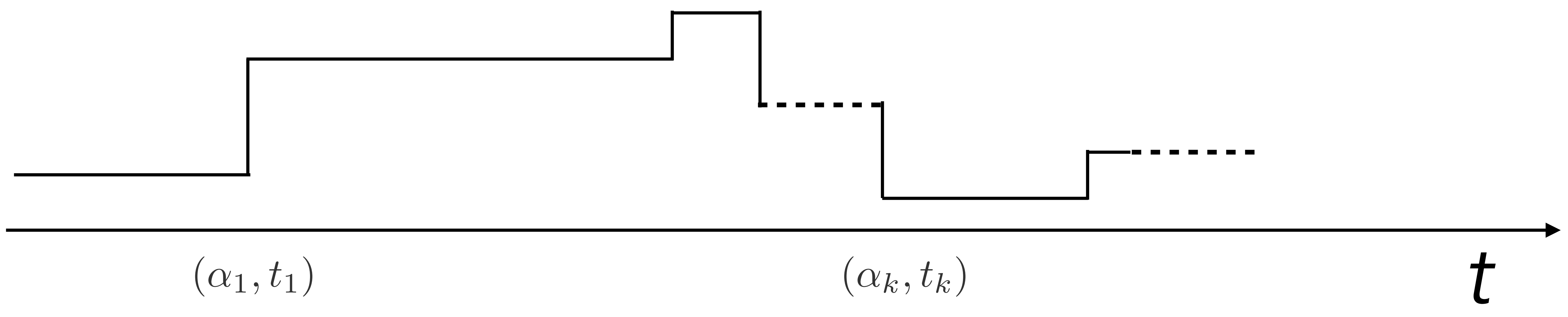
$$\sum_{\alpha=0} \hat{K}_\alpha^\dagger \hat{K}_\alpha = \hat{\mathbb{I}}$$

---

*Monitored dynamics*

Evolution $$|\psi'\rangle = \frac{\hat{K}_\alpha |\psi\rangle}{\sqrt{\langle \hat{K}_\alpha^\dagger \hat{K}_\alpha \rangle}}$$

with probability $p_\alpha = \langle \hat{K}_\alpha^\dagger \hat{K}_\alpha \rangle$

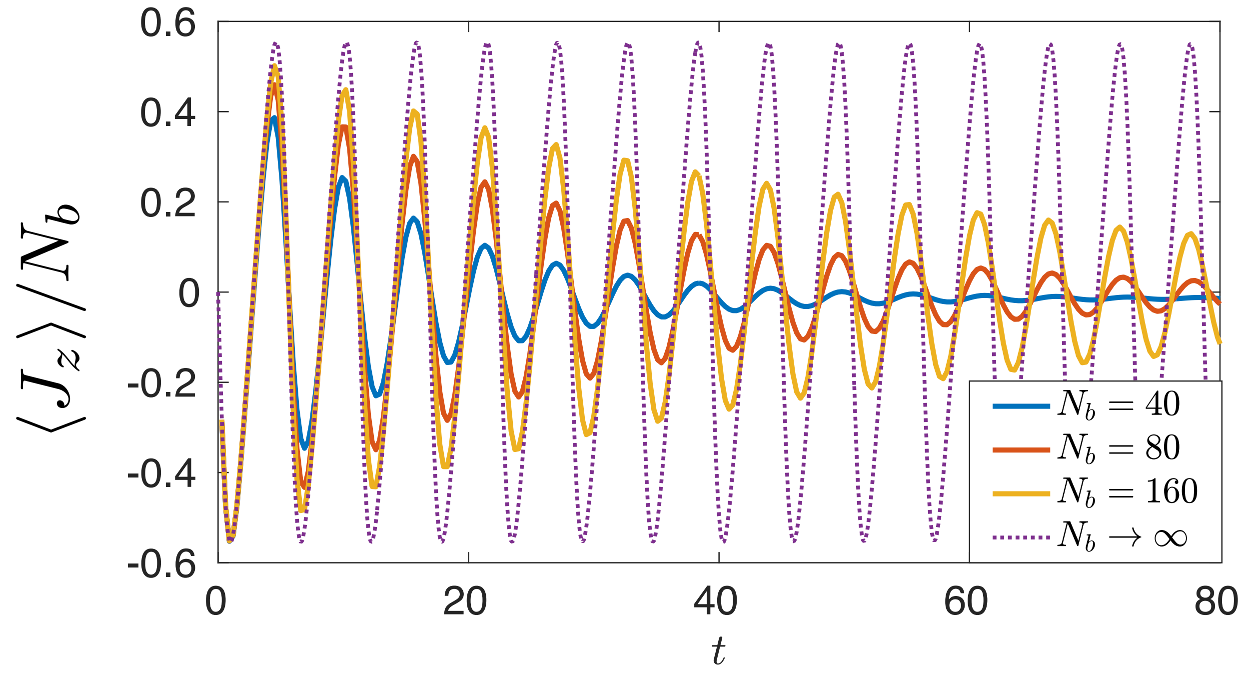


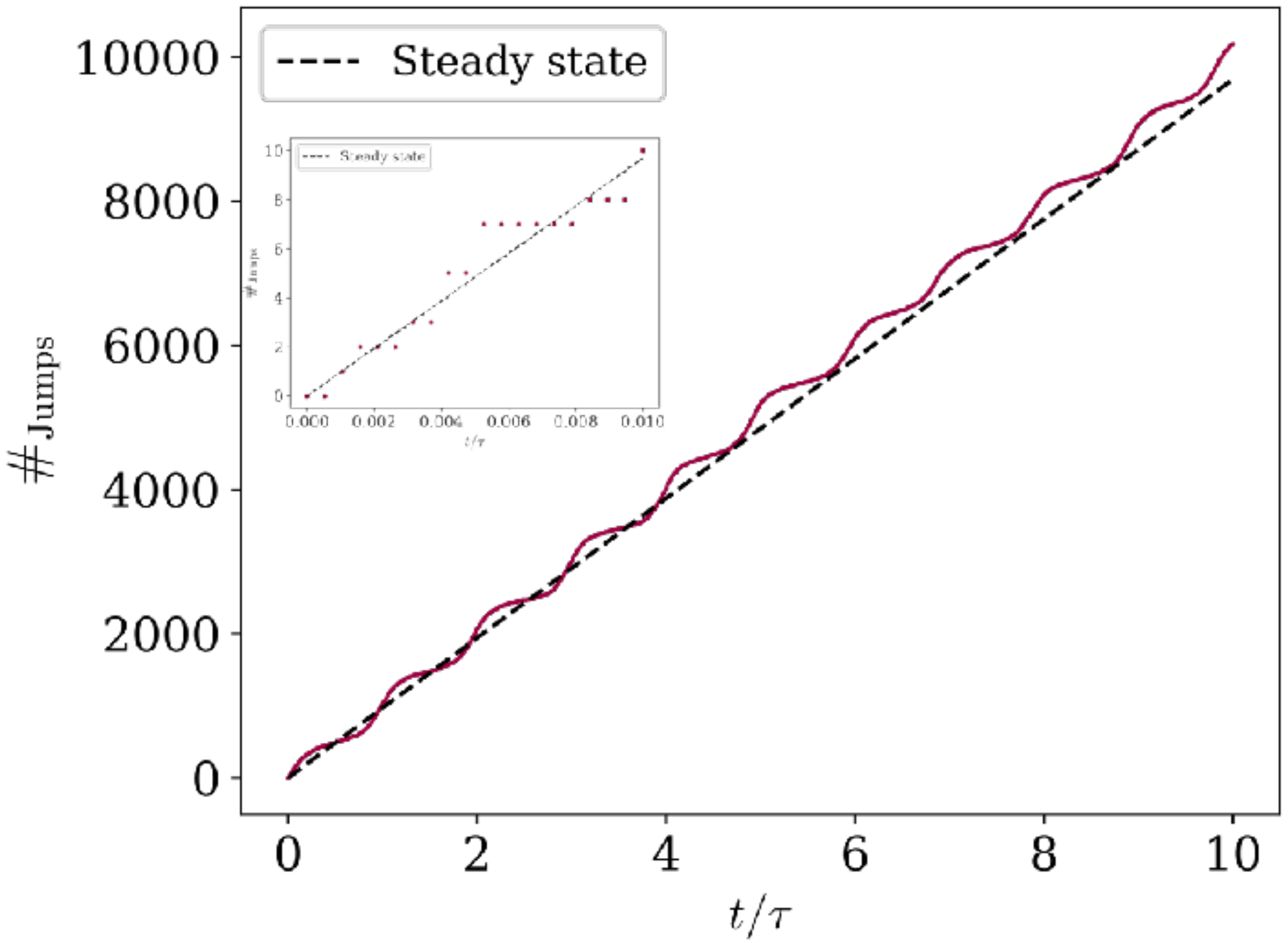

$$\mathcal{R}(t) = [(\alpha_1, t_1), (\alpha_2, t_2), \ldots (\alpha_k, t_k), \ldots]$$

# Collective dynamics of quantum jumps

Down jumps for our choice of unraveling …

Quantum jumps (from now on only down jumps) have a collective behaviours with statistical properties that reflect the existence of time crystal



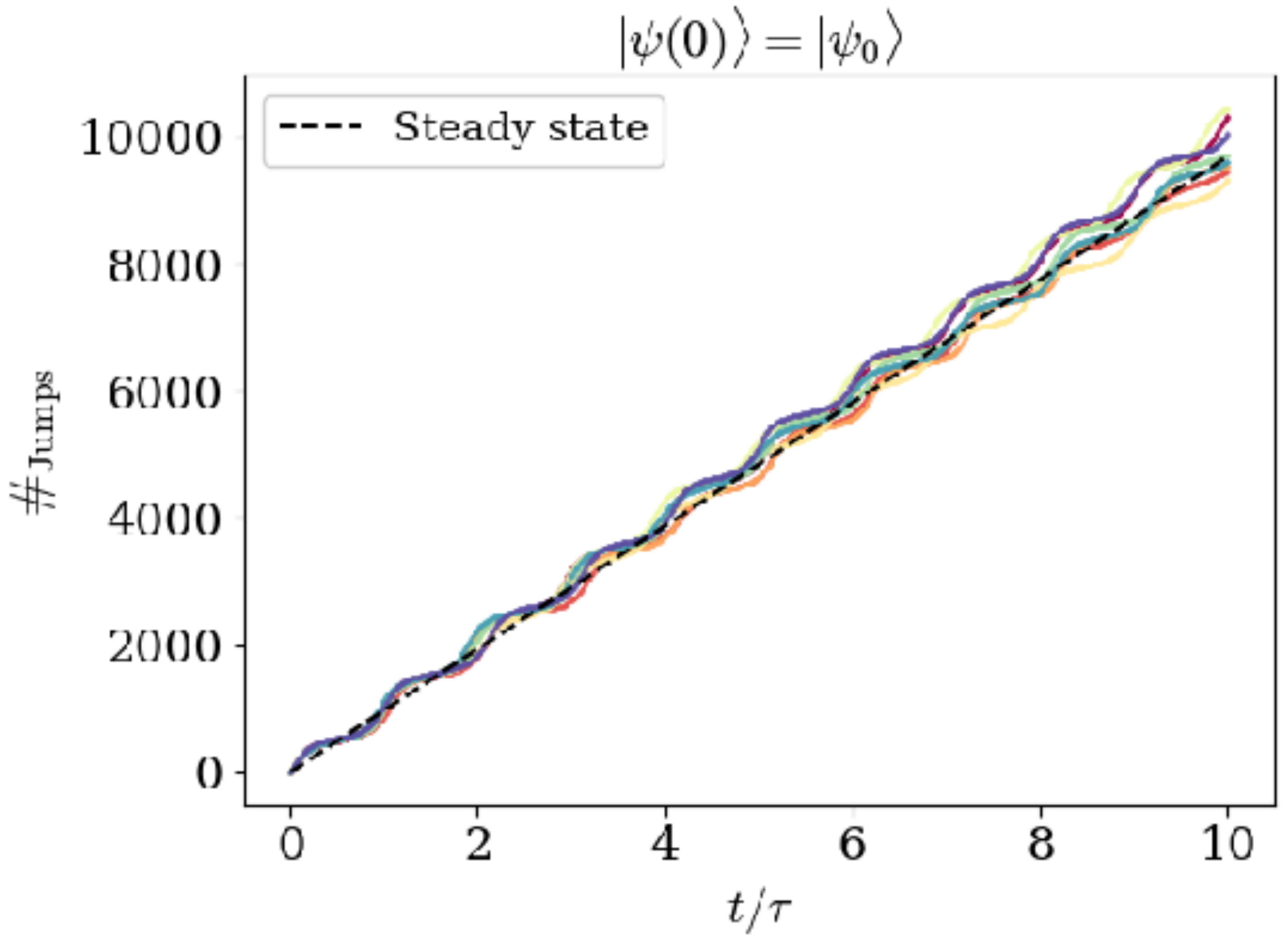

Jumps occurs in bursts separated by quite inert time intervals

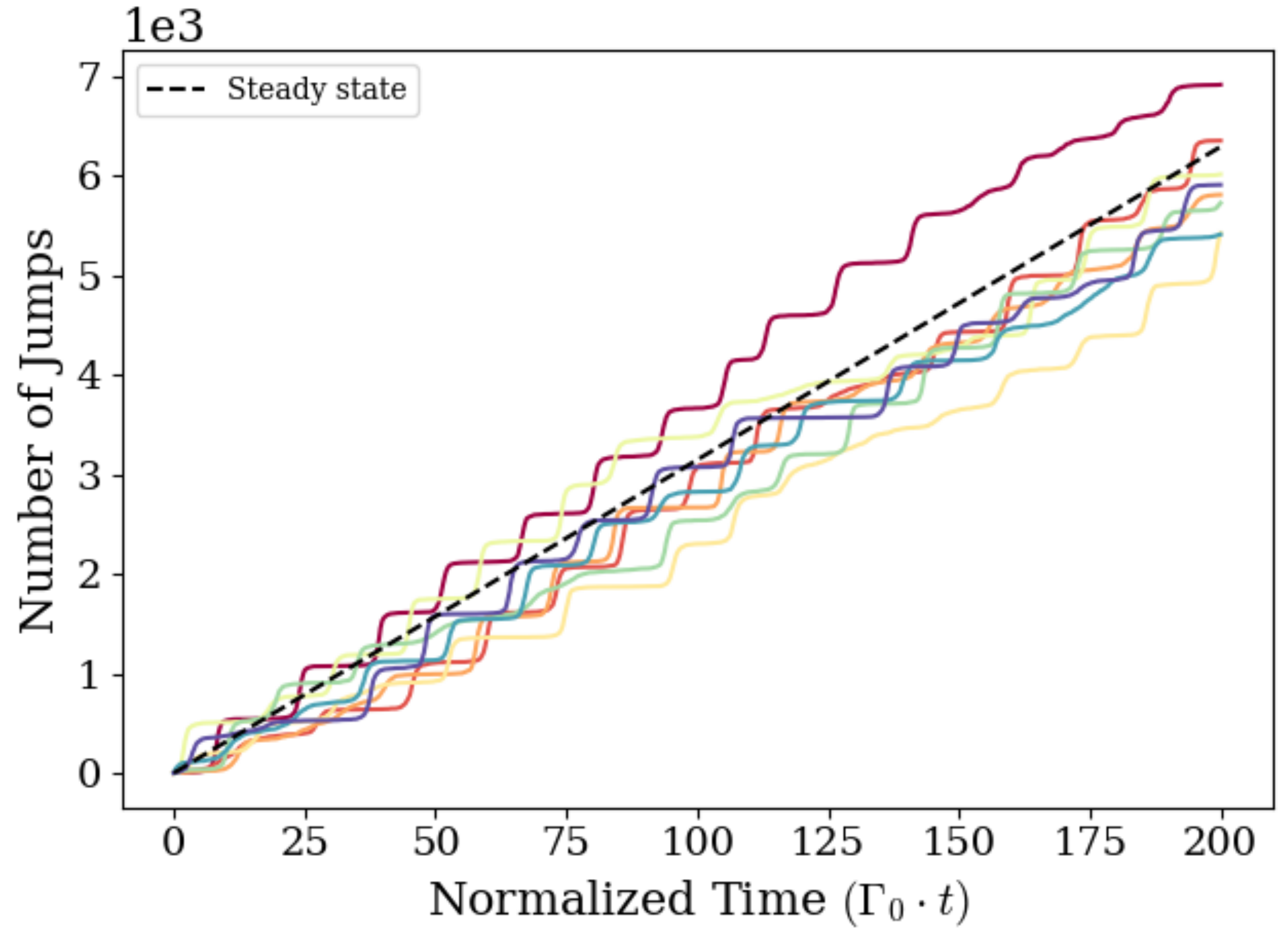
This behaviours (at finite number of spins) extends for times much longer than the typical decay time of the oscillations in the magnetisation.

# Collective dynamics of quantum jumps

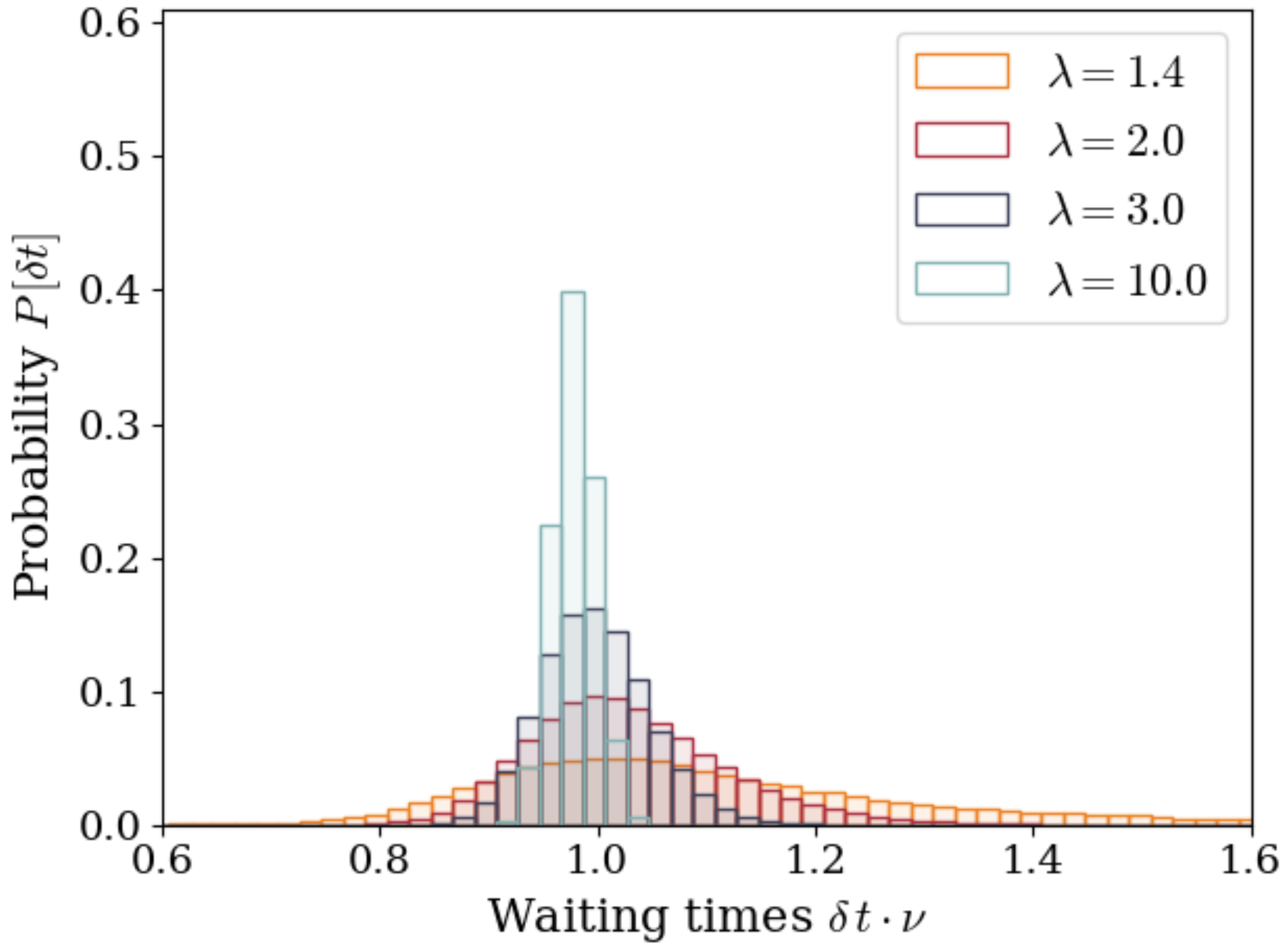
$\lambda = \Omega/\Gamma_0$

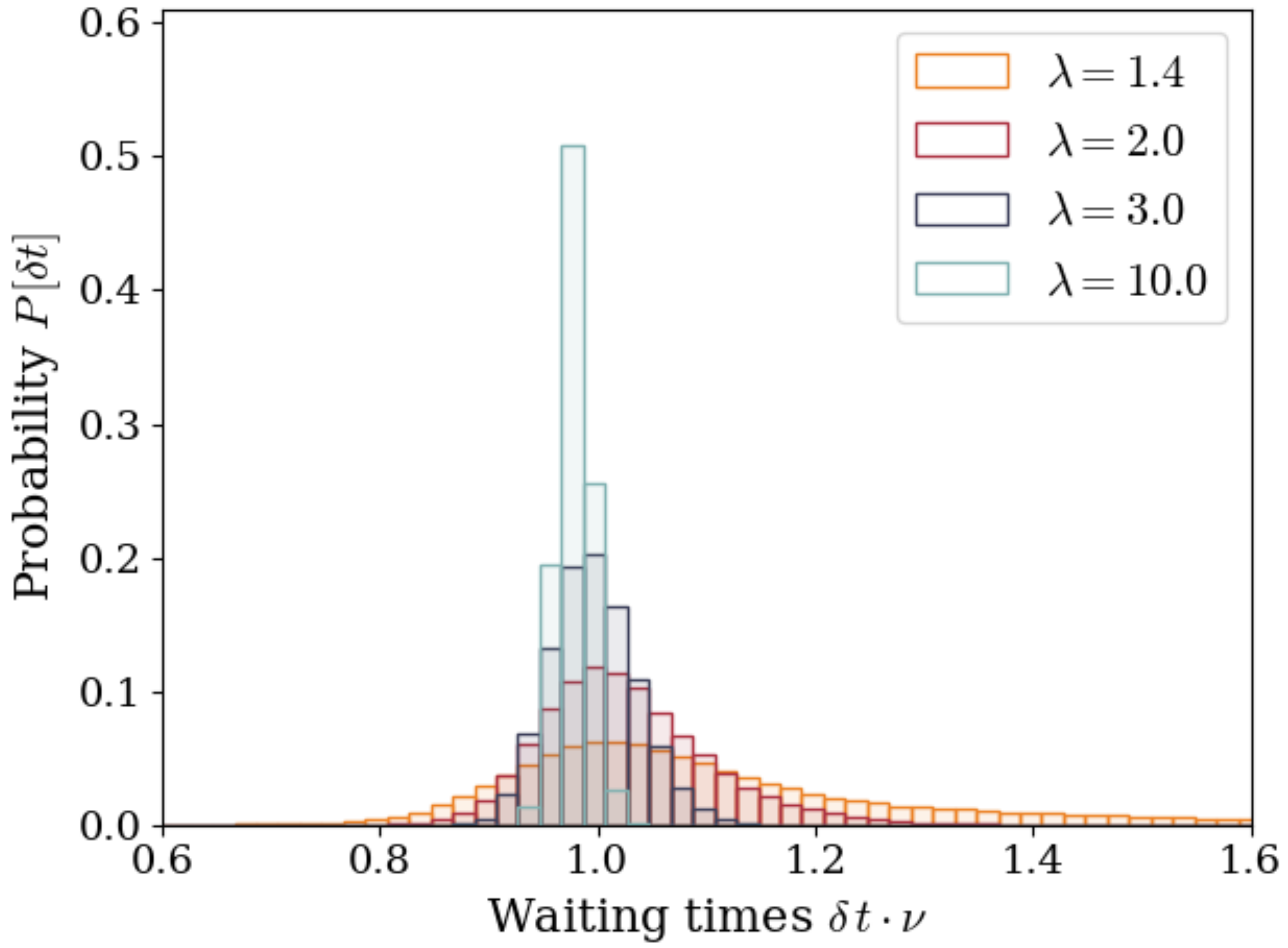
## Different trajectories phase-shift with time




## Waiting-time distribution




For fixed N, the distribution gets sharper going deeper in the time crystalline phase.

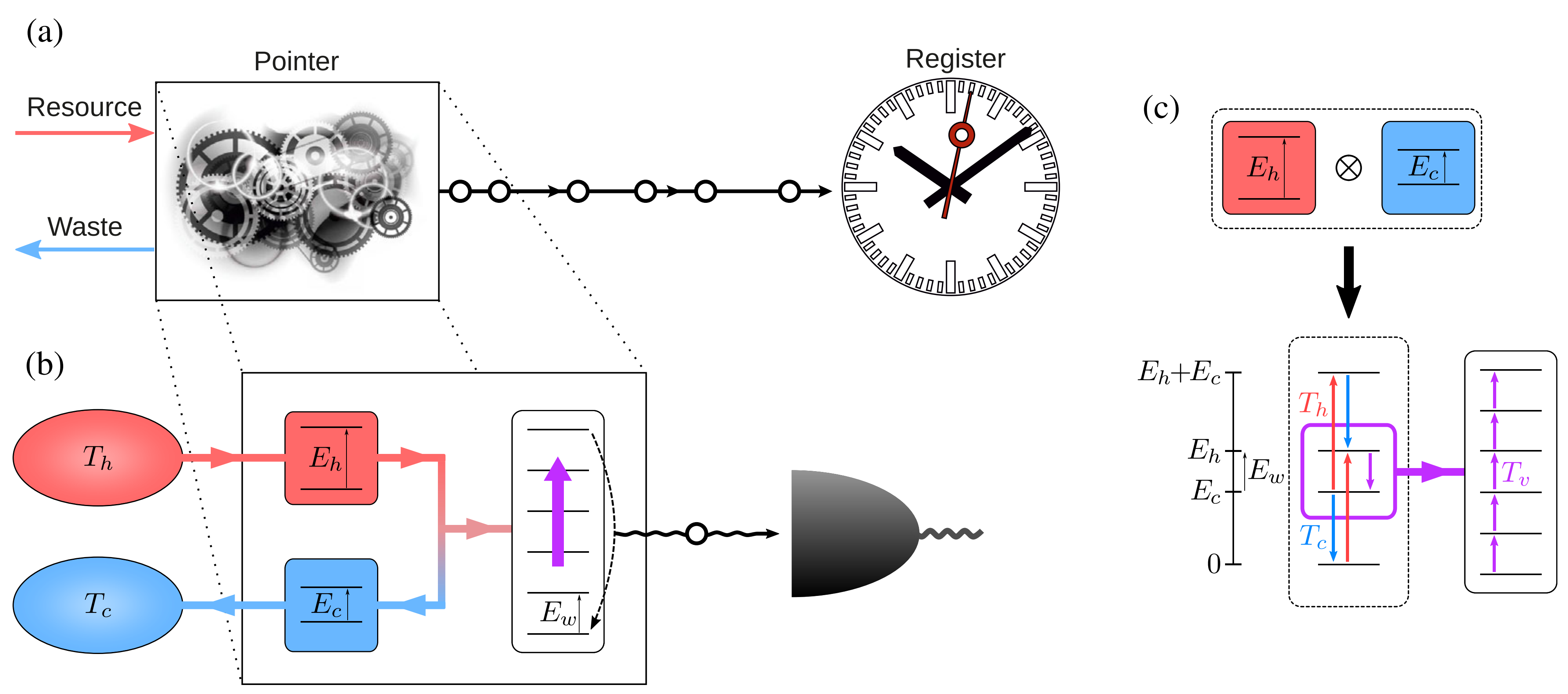
The mean value of the waiting times shows a weaker dependence on the coupling.

When comparing systems of different sizes at fixed coupling, distributions get sharper on increasing the size of the system.

# Thermodynamics of autonomous clocks

Autonomous clocks do not require any time-dependent control that would necessitate another external clock.

PAUL ERKER *et al.* PHYS. REV. X **7,** 031022 (2017)

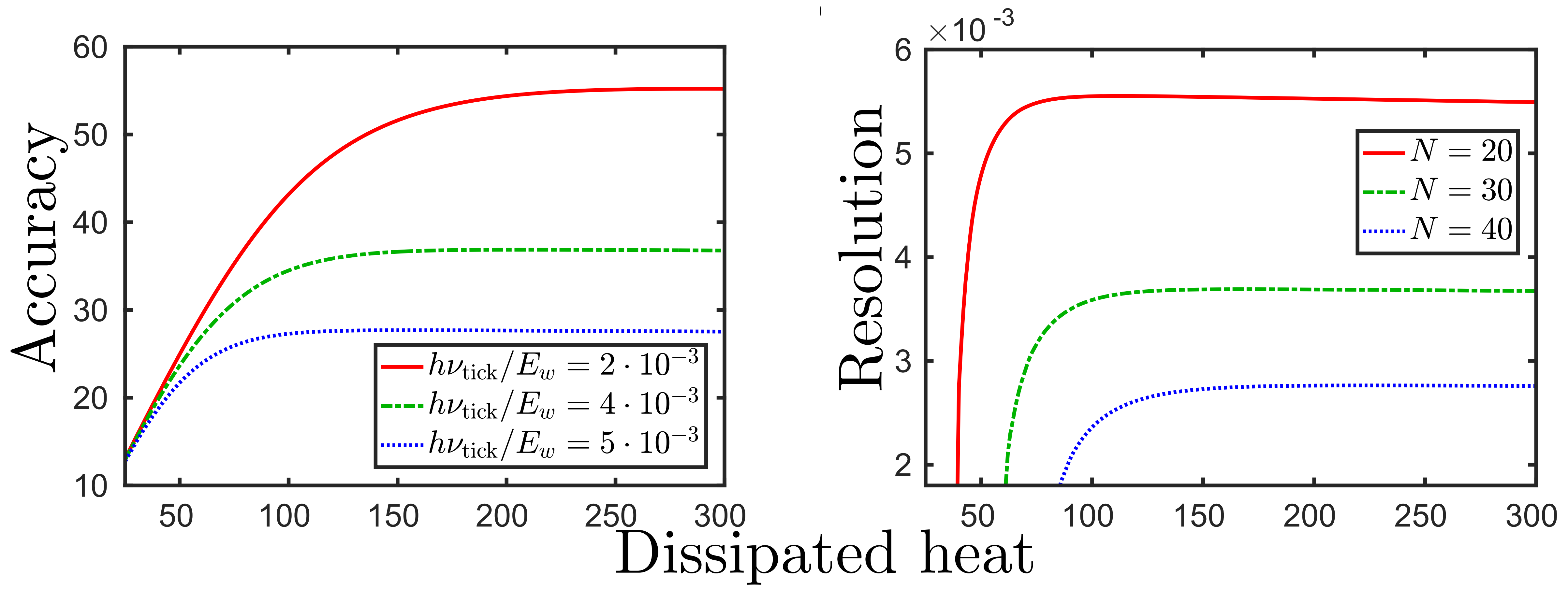



Autonomous clocks operate out of thermal equilibrium (simple example: a clock–powered by two thermal baths at different temperatures) T

The laws of thermodynamics dictate a trade-off between the dissipated heat and the clock's performance.

**A clock:** continuously provides a time reference to an external observer.

**Clock as a bipartite system:** composed by a pointer and a register, which stores classical information and transfer the information to an external observer. The pointer produce a sequence of signals, which are recorded by the register as ticks.



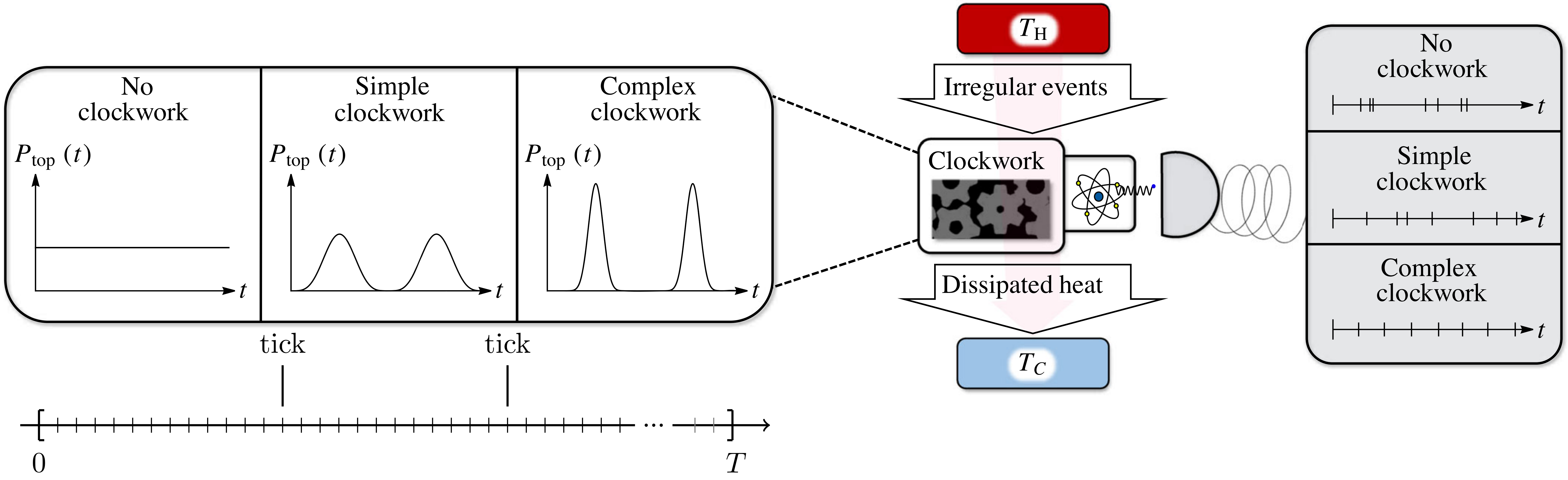

There are fundamental costs associated with accurate and precise timekeeping

Increasing the accuracy and/or the resolution requires a larger amount of entropy

# Thermodynamics of autonomous clocks

Emanuel Schwarzhans,[1,*] Maximilian P. E. Lock,[1] Paul Erker,[1] Nicolai Friis,[1] and Marcus Huber[1,2,†]

AUTONOMOUS TEMPORAL PROBABILITY CONCENTRATION: … PHYS. REV. X **11**, 011046 (2021)



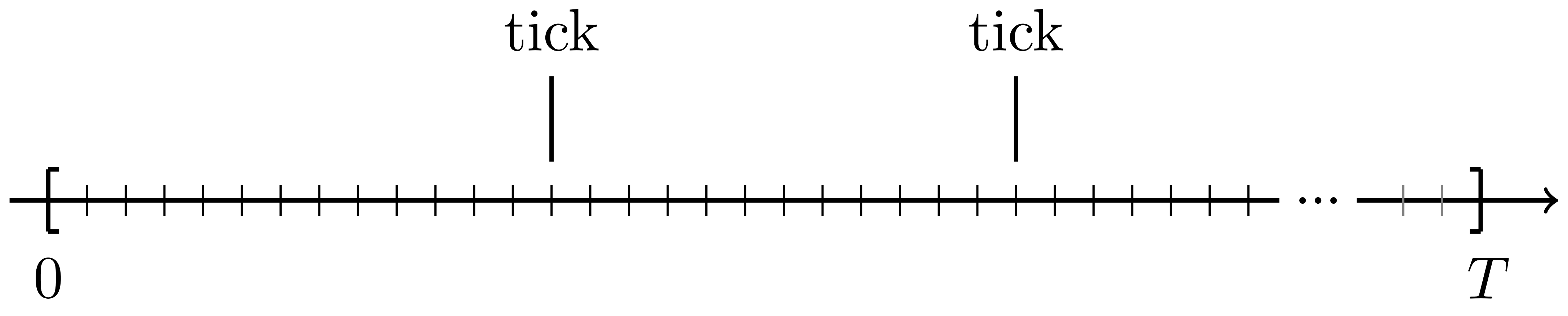

The purpose of a clock is to produce a regular sequence of ticks out of an irreversible process (that leads to an increase of entropy)

One should increase the complexity of the clockwork in order to induce the temporal probability concentration

Is a time crystal a potential good clock?

# Thermodynamics of autonomous clocks

## Figures of Merit




**resolution**: how frequently the clock ticks. It is defined as the inverse of the mean (both over each trajectory and over the ensemble of trajectories) waiting time $\tau$

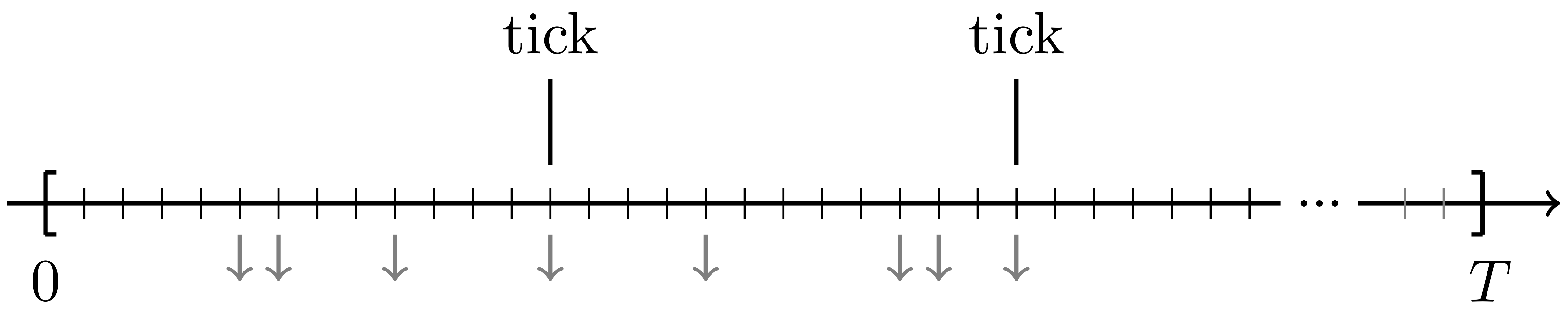
**accuracy**: is a measure of the relative dispersion of the waiting times, indicating how many ticks the clock provides before its uncertainty becomes greater than the average time between ticks.

$$\text{Accuracy} = \left(\frac{\tau}{\sigma_\tau}\right)^2$$

**Fano factor**: The Fano Factor of the clock can be written as the inverse of the product resolution × Accuracy.

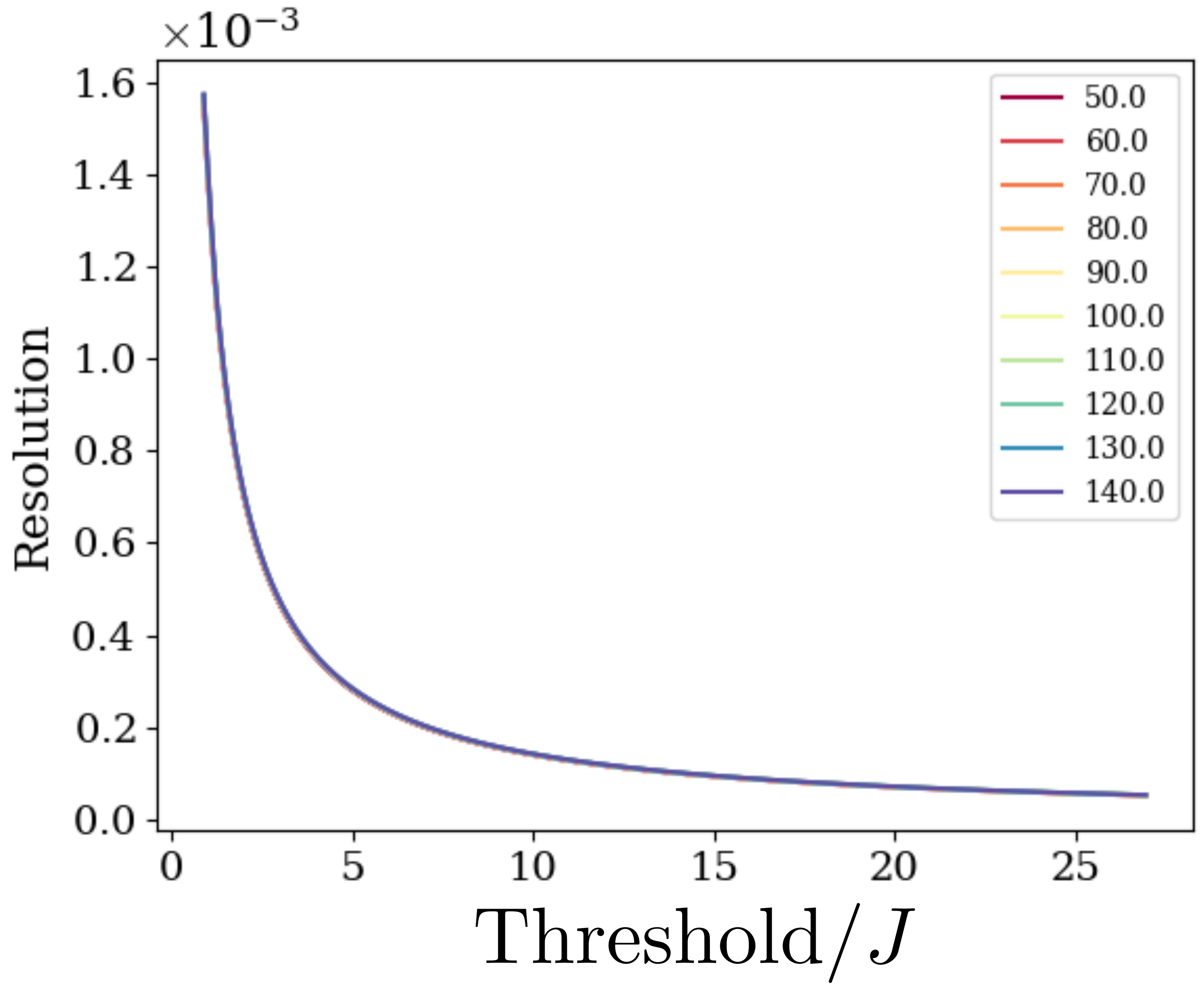
$$\text{Fano Factor} = \frac{\sigma_\tau^2}{\tau}$$

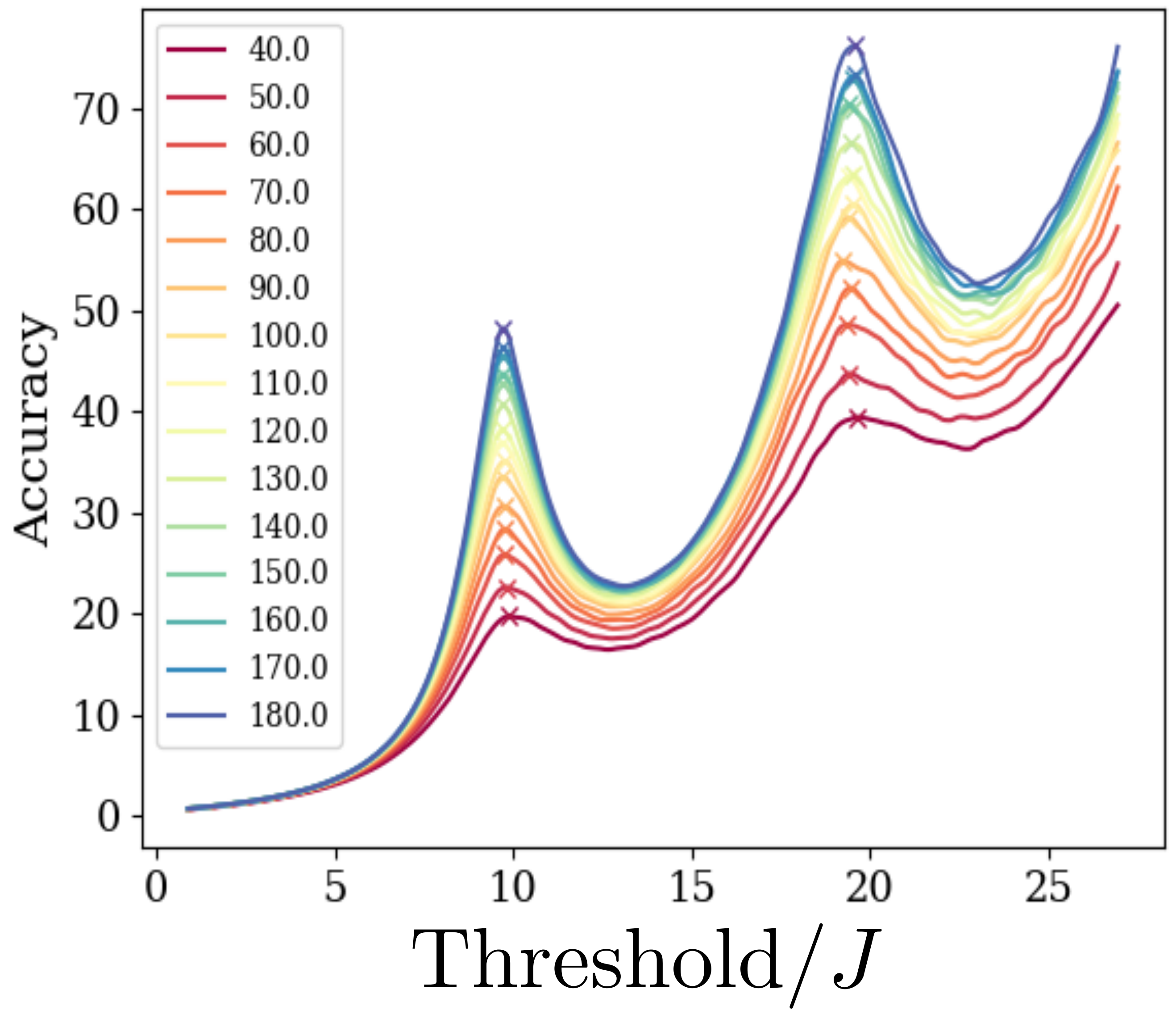
# Time-crystal as a clock

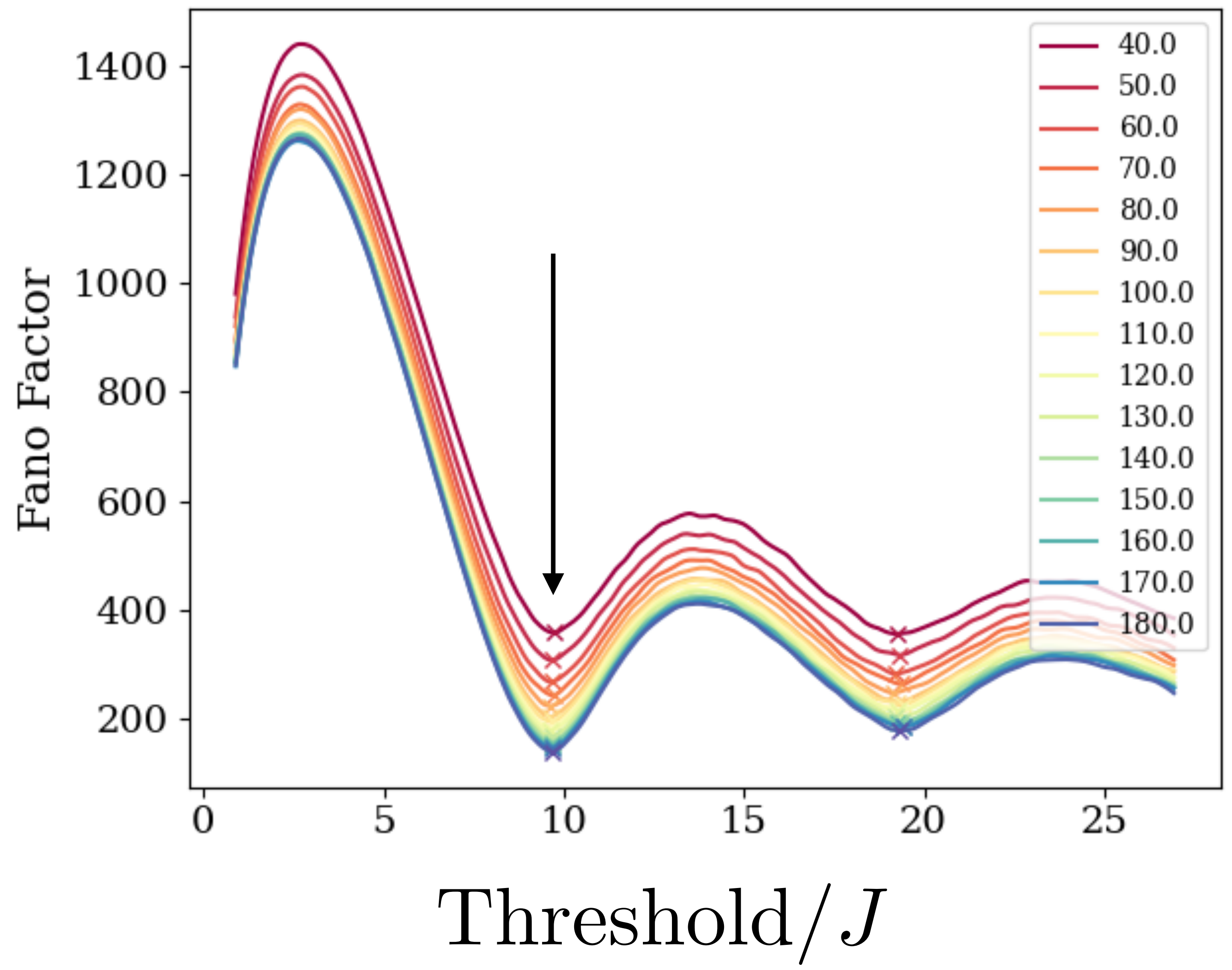



Down-jumps are used as ticks. It is useful to introduce a threshold that gives a tick after a certain amount of quantum jumps.

The figures of merit will depend on the choice of the threshold



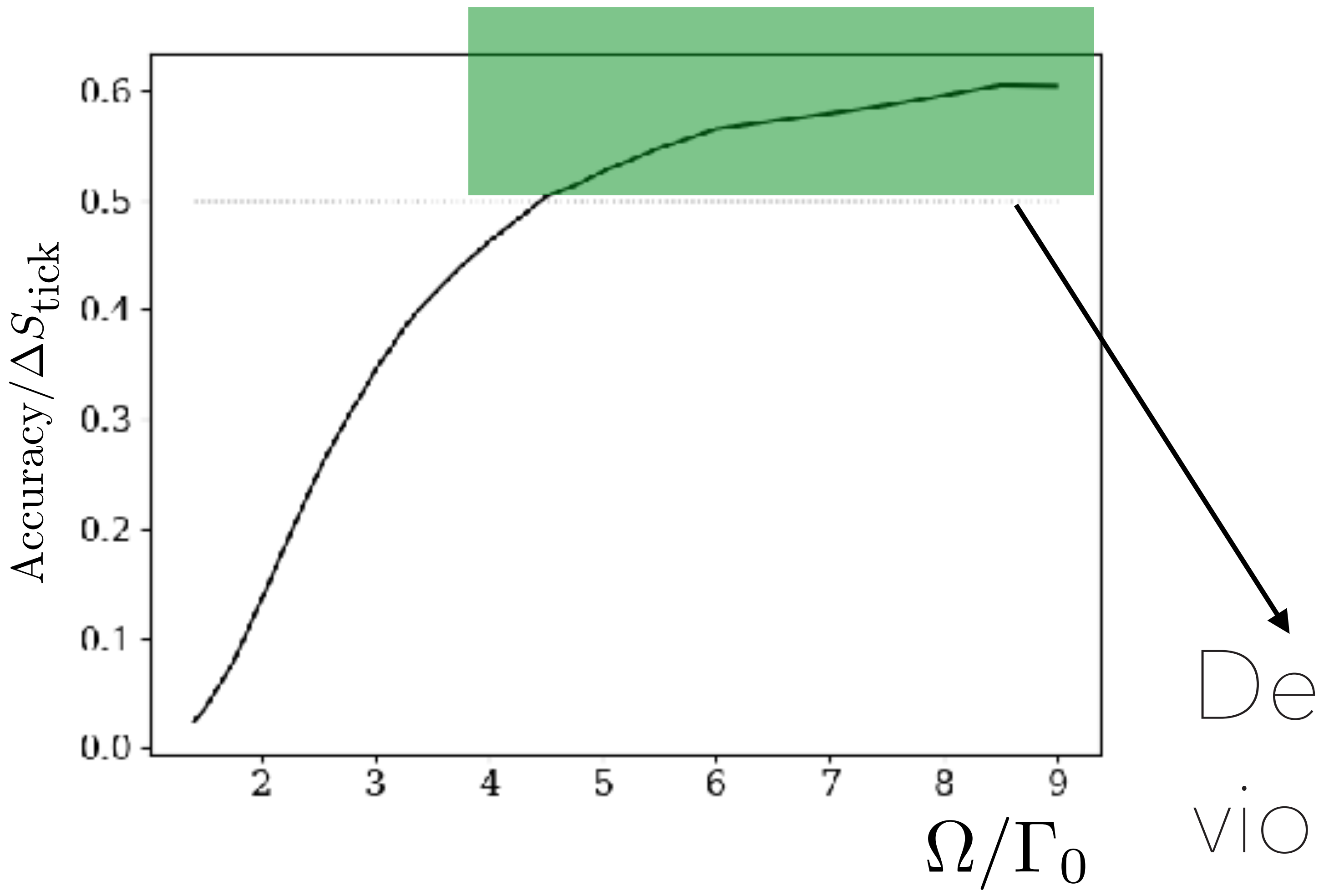




While the resolution is featureless and simply decreases with the choice of the threshold, the accuracy shows non-trivial dependence associated to the collective dynamics of jumps.

Existence of an optimal threshold

# Time-crystal as a clock




Accuracy limited by entropy production with bound connected to thermodynamics uncertainty relations (quantum systems may violate these bounds)

$$\frac{\text{Accuracy}}{\Delta S_{\text{tick}}} \leq \frac{1}{2}$$

Deep in the time-crystal phase the accuracy violates the bound

# Quantum Sensing

(i) the initialization of the sensor in an "advantageous/entangled" state;

(ii) a time interval in which the sensor interacts with the signal of interest (in our case h), so that the unknown parameter is encoded in the state of the sensor;

(iii) a measurement on the quantum sensor. By collecting the statistics of the repeated protocol, one infers the value of the parameter with maximal accuracy.

The least uncertainty on the estimated parameter is settled by the quantum Cramer-Rao bound

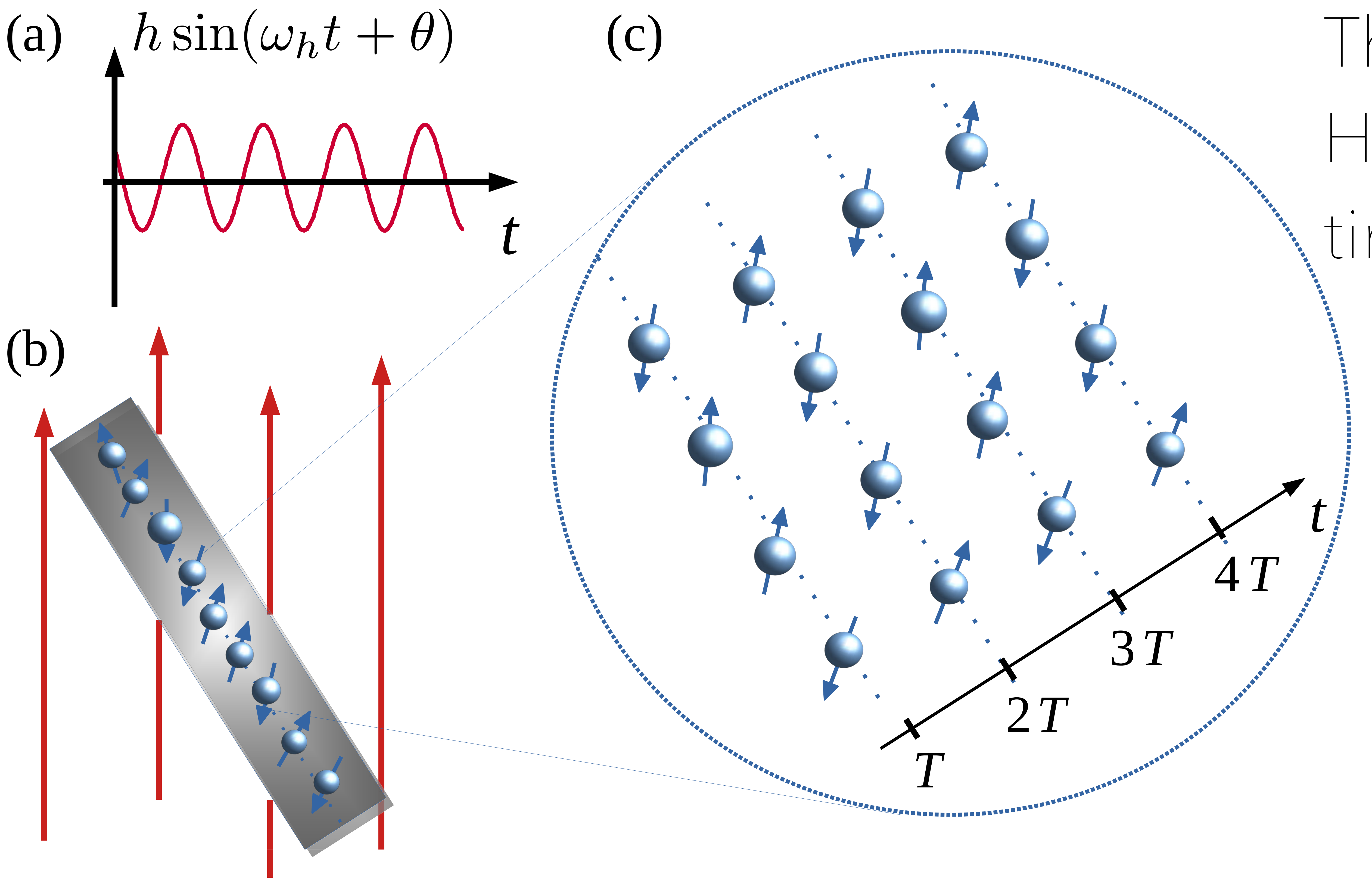
$$\Delta h(t) \geq \frac{1}{\sqrt{M F_h(t)}}$$

Quantum Fisher information (that for pure states is)

$$F_h(t) = 4\langle \partial_h \psi(t,h) | \partial_h \psi(t,h) \rangle - 4 |\langle \psi(t,h) | \partial_h \psi(t,h) \rangle|^2$$

The quantum Fisher information provides the ultimate lower bound to the achievable uncertainty for optimized quantum measurements

# The Model




The quantum sensor is described by the Hamiltonian $\hat{H}_s$ which is coupled for a given time to the signal $\hat{V}(t)$

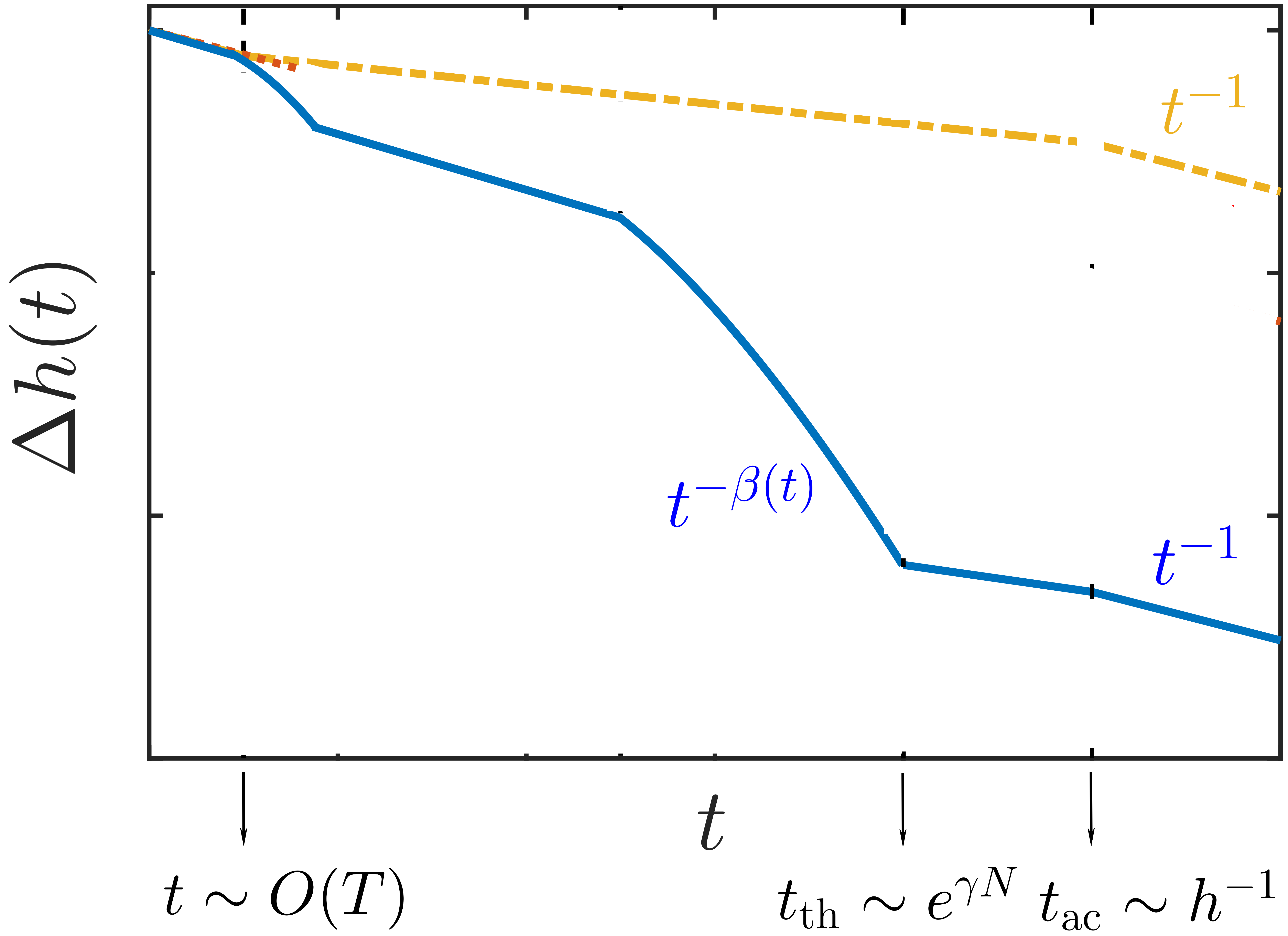
The signal is an AC field whose amplitude we want to measure

$$\hat{V} = \frac{h}{2}\sin(\omega_h t + \theta)\sum_i \hat{\sigma}_i^z$$

The sensor is described by a Floquet Hamiltonian that can enter a TC phase

$$\hat{H}_s = \sum_i \left[ J_i \hat{\sigma}_i^z \hat{\sigma}_{i+1}^z + \sum_{\alpha=x,z} b_i^\alpha \hat{\sigma}_i^\alpha - \frac{\phi}{2}\sum_{n=-\infty}^{\infty} \delta(t - nT)\hat{\sigma}_i^x \right]$$

# MBL Quantum Sensor




**SQL**: independent spins

**Ergodic phase**: correlated spins, non-resonant

Note: No claim of optimality

**MBL Quantum Sensor:** it takes advantage from i) resonant condition, ii) (moderate) presence of quantum correlations, iii) exp-large interrogation time

# Dissipation and sensing

- Here we considered the case in which there is no external noise. This will strongly affect MBL and the existence of time-crystals.

- The presence of an external environment is compatible with time crystals (dissipative/continuous/boundary TCs)

*Quantum enhancements and entropic constraints to Boundary Time Crystals as sensors of AC fields*

Dominic Gribben, Anna Sanpera, R. F., Jamir Marino, Fernando Iemini
arXiv:2406.06273

# Conclusions

- Time crystals can be used as autonomous clocks
- Is the behaviour generic of dissipative TCs ?
- I discussed Floquet TCs as quantum sensors for AC-fields.
- Their optimal performance offer several advantages, overcoming the SQL, allowing long-time sensing measurements times exponentially large with the number of spins

Thank you

European Research Council