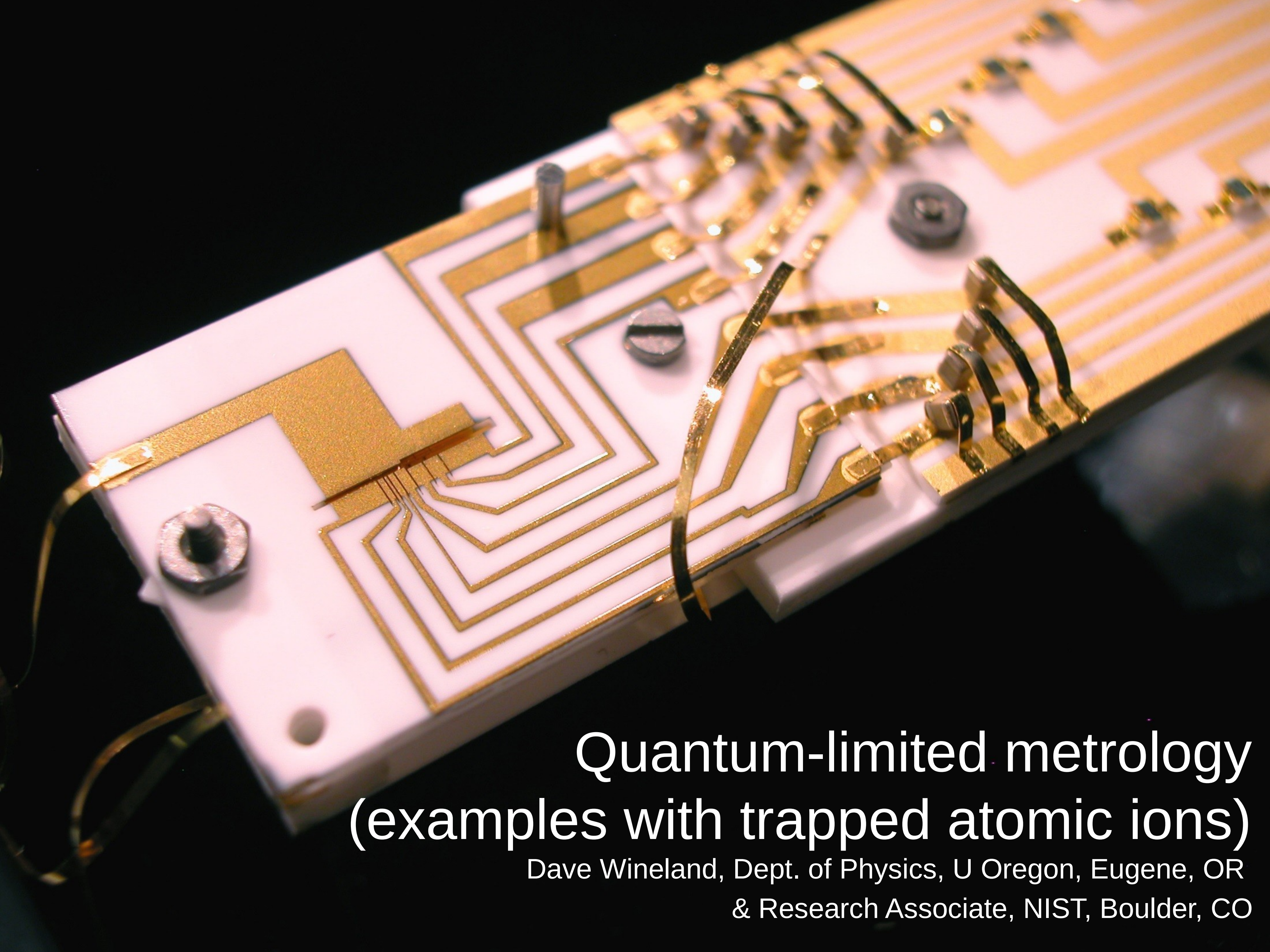


Quantum-limited metrology (examples with trapped atomic ions)

Dave Wineland, Dept. of Physics, U Oregon, Eugene, OR
& Research Associate, NIST, Boulder, CO

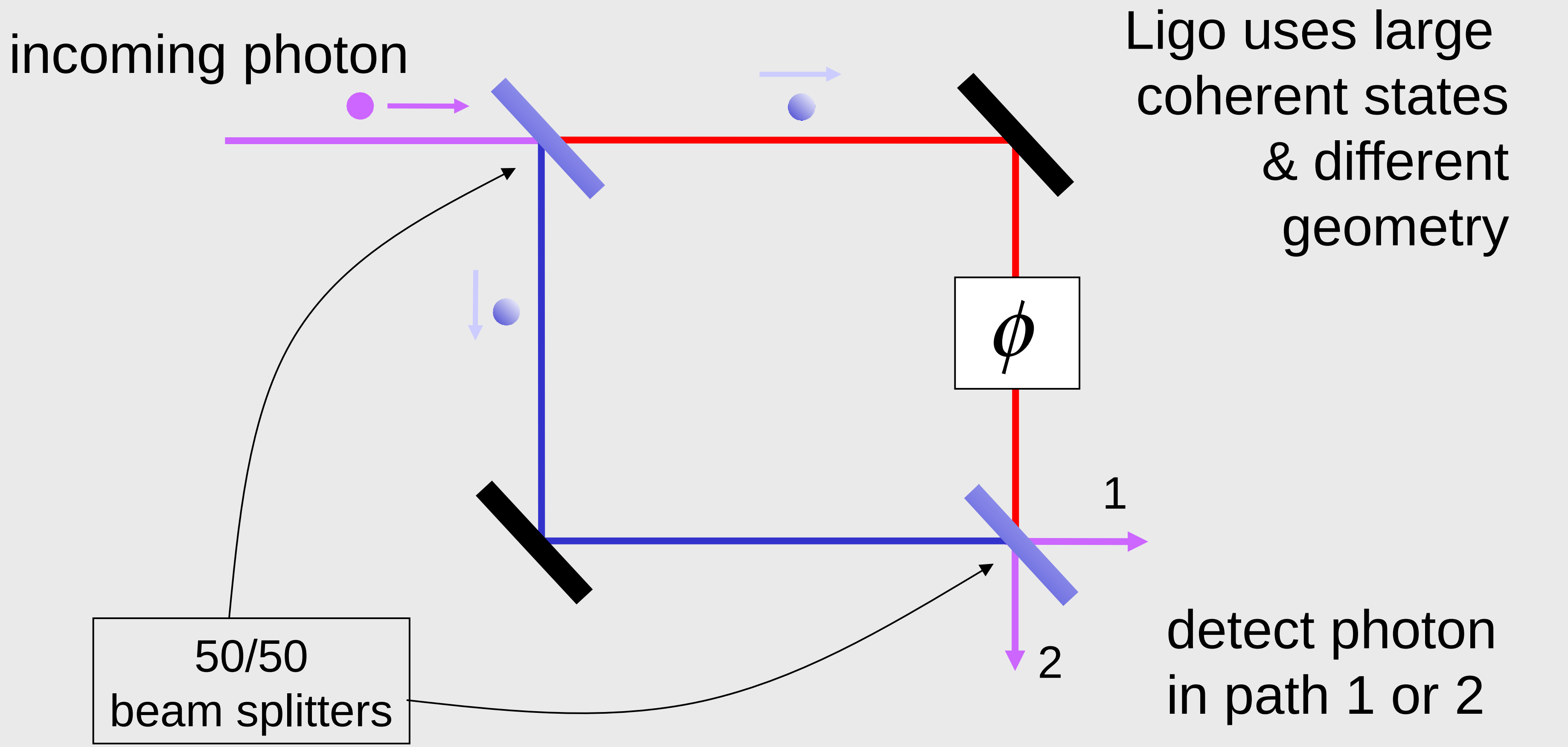
Summary:

- Ramsey interferometer, angular momentum picture
  * application to spectroscopy/clocks
- entangled states for increased precision
  * spin squeezing
  * spin "Schrödinger cat" states
- efficient detection with ancilla qubits
  * application to spectroscopy/clocks
- squeezed harmonic oscillator states

Quantum-limited metrology (examples with trapped atomic ions)

Dave Wineland, Dept. of Physics, U Oregon, Eugene, OR
& Research Associate, NIST, Boulder, CO

Mach-Zehnder interferometer:



Carl Caves, …

review: V. Giovannetti, Science **306**, 1330 (2004).

Ramsey interferometry with qubits (states $|\downarrow\rangle$ and $|\uparrow\rangle$, $E_\uparrow - E_\downarrow = \hbar\omega_0$)

π/2 pulses like 50/50 beam splitters in Mach-Zehnder interferometer

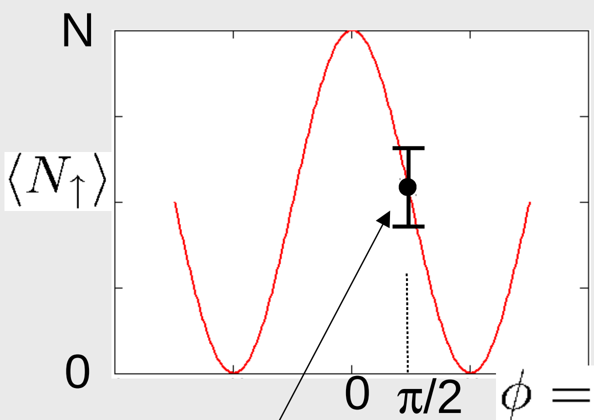
$$\Psi(t=0) = |\downarrow\rangle_1 \otimes |\downarrow\rangle_2 \cdots \otimes |\downarrow\rangle_N$$

applied radiation ("π/2 pulses")
frequency ω near ω_0

$$\rightarrow \frac{1}{2^{N/2}}\left[|\downarrow\rangle_1 + |\uparrow\rangle_1\right] \otimes \left[|\downarrow\rangle_2 + |\uparrow\rangle_2\right] \cdots \otimes \left[|\downarrow\rangle_N + |\uparrow\rangle_N\right]$$

measure

$$\langle N_\uparrow\rangle = NP_\uparrow$$



$$\phi = (\omega_0 - \omega)T \rightarrow$$

for each qubit: $P_\uparrow = \frac{1}{2}\left(1 + \cos\left(\omega - \omega_0\right)T\right)$

$$\langle N_\uparrow\rangle = NP_\uparrow = \frac{N}{2}(1 + \cos\phi)$$

"Signal" $\partial\langle \tilde{N}_\uparrow(t_f)\rangle/\partial\phi = -\frac{N}{2}\sin\phi$

Noise $\Delta\tilde{N}_\uparrow(t_f) = \sqrt{NP_\uparrow(1-P_\uparrow)} = \frac{\sqrt{N}}{2}\sin\phi$

$$S/N = \sqrt{N}$$

signal to noise independent of ϕ

$\Delta\tilde{O} \equiv (\langle(\tilde{O} - \langle\tilde{O}\rangle)^2\rangle)^{1/2}$

("projection noise")

Ramsey interferometer, angular momentum picture

$\mathbf{J} = \Sigma \mathbf{S}_i$ (S_i = ½, equivalent to ensemble of two-level systems)

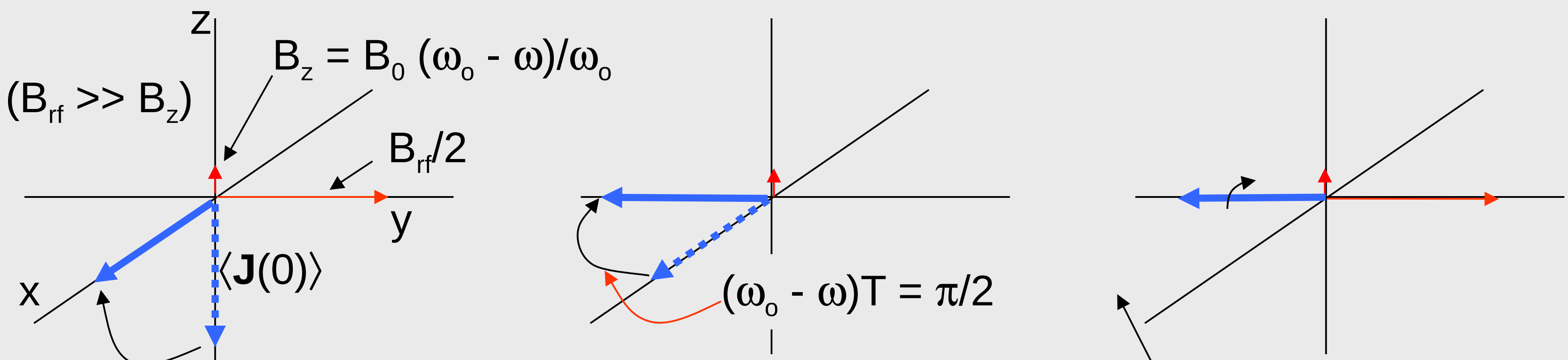
e.g., $H_i = \hbar\, \gamma S_z B_0$

R. Feynman *et al.* J. Appl. Phys. **28**, 49 (1957))

$$\Psi(t=0) = |\downarrow\rangle_1 \otimes |\downarrow\rangle_2 \cdots \otimes |\downarrow\rangle_N$$

$J = N/2$, $m_J = -N/2$ ("coherent spin state")

(in rotating frame of applied field (frequency ω):



First Ramsey pulse

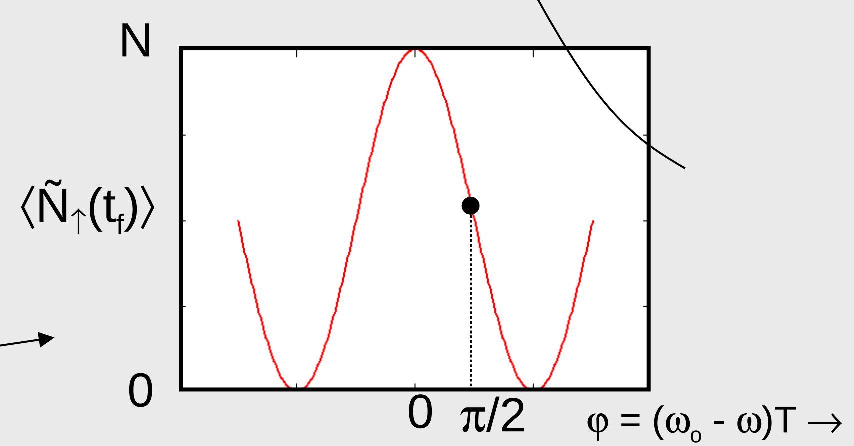
Free precession

Second Ramsey pulse

$$\tilde{O} = \tilde{N}_\uparrow(t_f) = \hat{J}_z + J\hat{I}$$

$$\langle \tilde{N}_\uparrow(t_f) \rangle = N/2(1 + \cos(\omega_o - \omega)T)$$

$(t_f = T)$



Measurement uncertainty relations:

Uncertainty relation for operators:

For operators $\tilde{O}_1$, $\tilde{O}_2$, Schwartz inequality

(1) $\Delta\tilde{O}_1\ \Delta\tilde{O}_2 \geq ½|\langle[\tilde{O}_1,\tilde{O}_2]\rangle|$ (measurement fluctuations on identically prepared systems)

for $\tilde{O}_1 = x$, $\tilde{O}_2 = p$, Eq. (1) gives position/momentum uncertainty relation

Time/Energy uncertaintly relation:

Uncertainty relations for parameters and operators

For operator $\tilde{O}(\xi)$ (ξ = parameter)

$\Delta\tilde{O} \cong |d\langle\tilde{O}\rangle/d\xi|\Delta\xi, \quad \Rightarrow \Delta\xi \cong \Delta\tilde{O}\ /|d\langle\tilde{O}\rangle/d\xi|$

In (1), let $\tilde{O}_1 = \tilde{O}$, $\tilde{O}_2 = \mathbf{H}$ (Hamiltonian) $\Rightarrow \Delta\tilde{O}\Delta\mathbf{H} \geq ½|\ \langle[\tilde{O},\mathbf{H}]\rangle|$

But, $\hbar\ d\tilde{O}/dt = i[\mathbf{H},\tilde{O}] + \hbar\ \partial\tilde{O}/\partial t \Rightarrow \Delta\tilde{O}\ \Delta\mathbf{H} \geq\ ½\ \hbar\ |d\langle\tilde{O}\rangle/dt|$ (for $\partial\tilde{O}/\partial t = 0$)

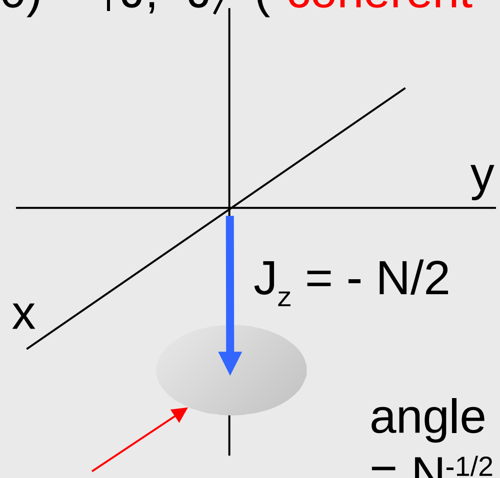
$\Delta\tilde{O} = |d\langle\tilde{O}\rangle/dt|\ \Delta t, \ \Rightarrow \Delta H\ \Delta t \geq ½\ \hbar$

Quantum limits to (angular momentum) rotation angle measurement

For J(0) = $|J, -J\rangle$ (“coherent” spin state)

Uncertainty relation:

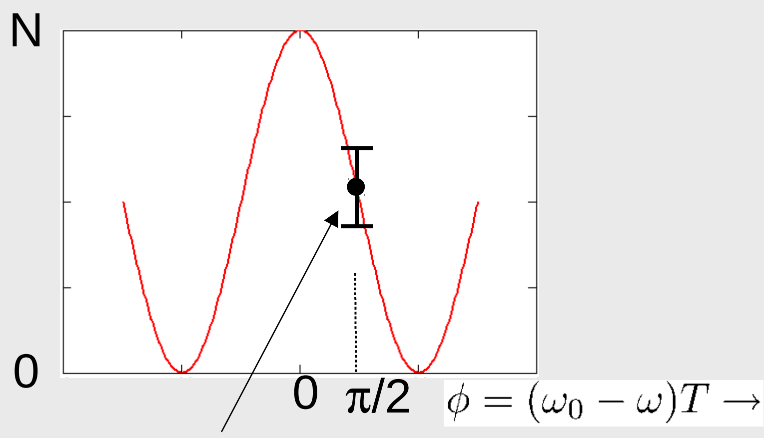
$$\Delta J_x \cdot \Delta J_y \geq \tfrac{1}{2}\left|\left\langle \left[J_x, J_y \right] \right\rangle\right| = \tfrac{1}{2}\left|\left\langle J_z \right\rangle\right|$$



angle sensitivity
= $N^{-1/2}$

$$\Delta J_z(0) = 0,$$

$$\Delta J_x(0) = \Delta J_y(0) = \sqrt{J/2} = \tfrac{1}{2}\sqrt{N}$$



phase sensitivity $\Delta\phi = N^{-1/2}$

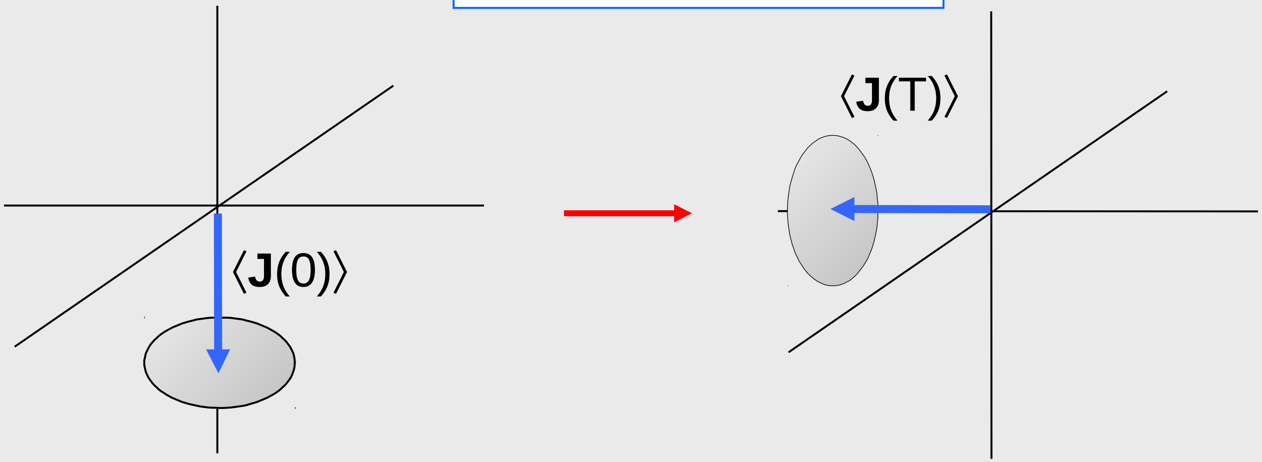
<u>independent of</u> ϕ

<u>observed for coherent spin states:</u>

- Itano *et al.*, PRA <u>47</u>, 3554 (1993).
- Santarelli *et al.*, PRL **82**, 4619 (1999).
- …

“projection noise”

coherent spin state

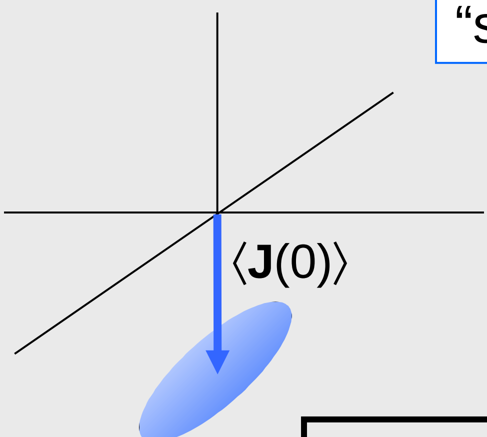


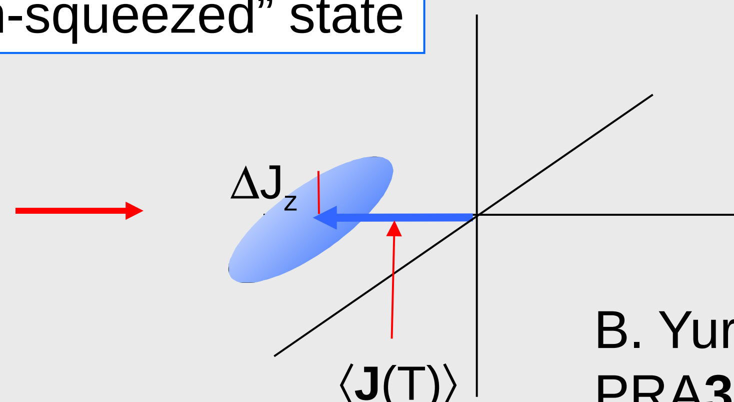
angle sensitivity
$= N^{-1/2}$

WANT:

$$\varphi = (\omega_o - \omega)T$$

“spin-squeezed” state

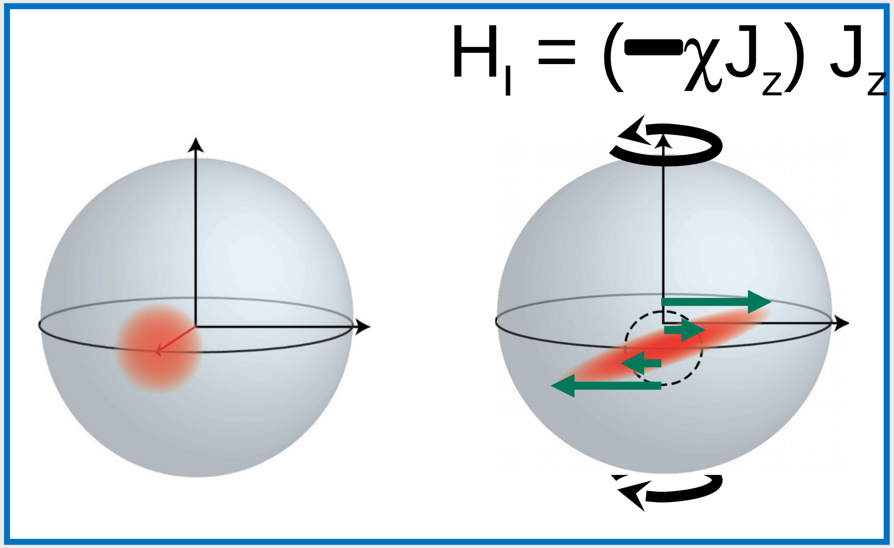




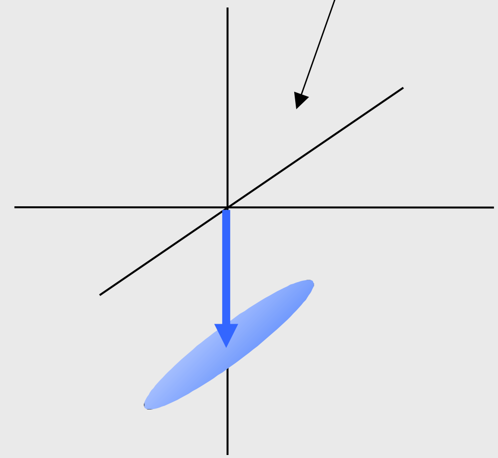
B. Yurke *et al*.,
PRA**33**, 4033 (1986)

$$\text{Angle sensitivity} = \frac{\Delta J_\perp}{\langle \mathbf{J}(T) \rangle}$$

Generate spin squeezing with $H_I = -\chi J_z^2$, $\Rightarrow U = \exp(-i\chi t J_z^2)$



test by first rotating state to



κ = signal-to-noise improvement $= \dfrac{\Delta\theta(\text{coherent})}{\Delta\theta(\text{squeezed})} = \dfrac{1/\sqrt{2J}}{\Delta \mathbf{J}_\perp / |\langle \mathbf{J} \rangle|}$

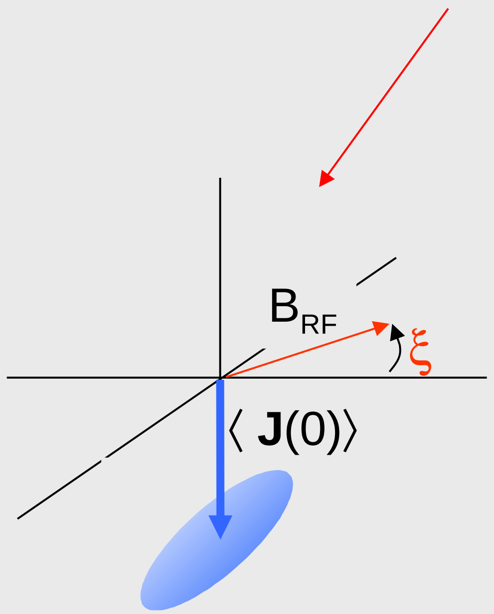
$$\kappa = \frac{|\langle \mathbf{J} \rangle|}{\Delta \mathbf{J}_\perp \sqrt{2J}}$$

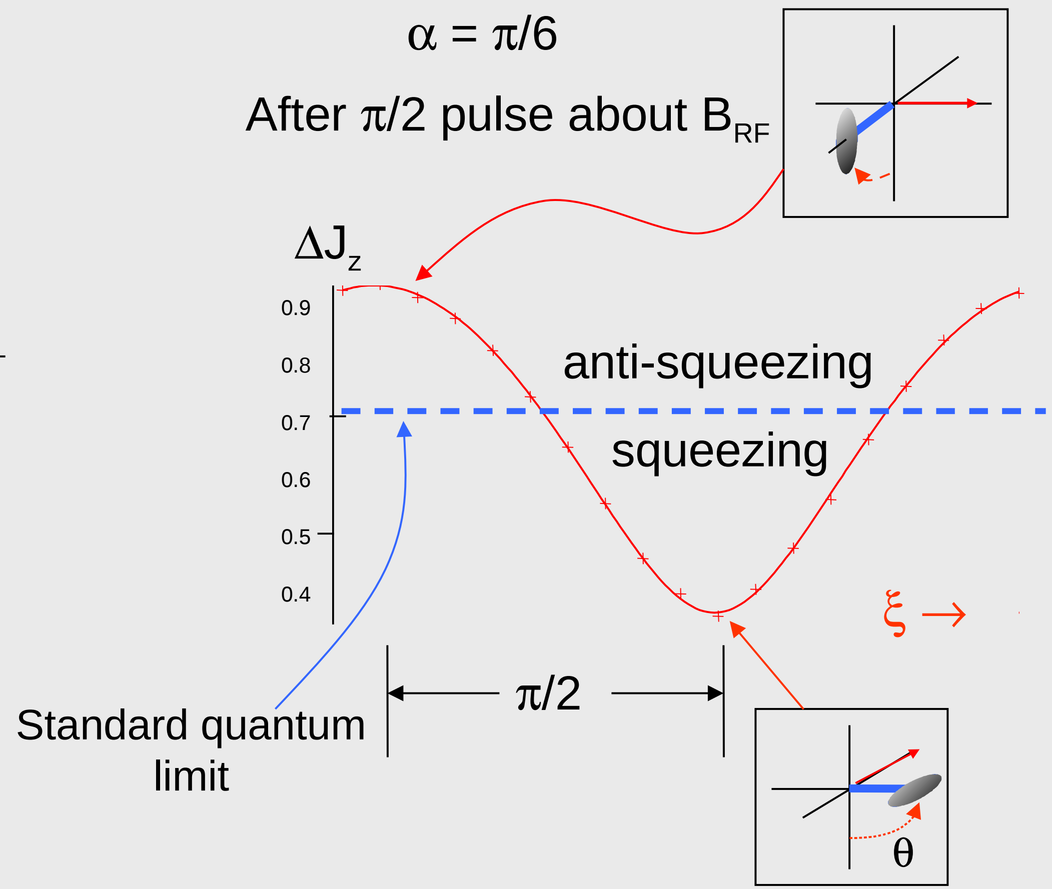
<u>Note:</u> $|\langle \mathbf{J} \rangle|$ shrinks with squeezing

- Sanders, Phys. Rev. A**40**, 2417 (1989)
  (nonlinear beam splitter for photons)
- Kitagawa and Ueda, Phys. Rev. A**47**, 5138 (1993)
  (potentially realized by Coulomb interaction in electron interferometers)
- Sørensen & Mølmer, PRL **82**, 1971 (1999) (trapped ions)
- Solano, de Matos Filho, Zagury PRA**59**, 2539 (1999) (trapped ions)
- Milburn, Schneider and James, Fortschr. Physik **48,** 801 (2000) (trapped ions)

apply $H_I \propto J_x^2$. For N = 2,

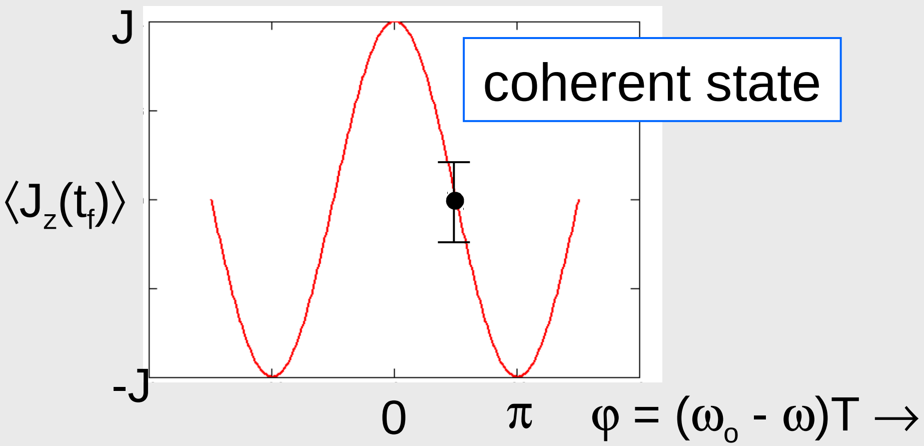
$$|\downarrow\rangle|\downarrow\rangle \rightarrow \cos(\alpha)|\downarrow\rangle|\downarrow\rangle + i\sin(\alpha)\,|\uparrow\rangle|\uparrow\rangle$$

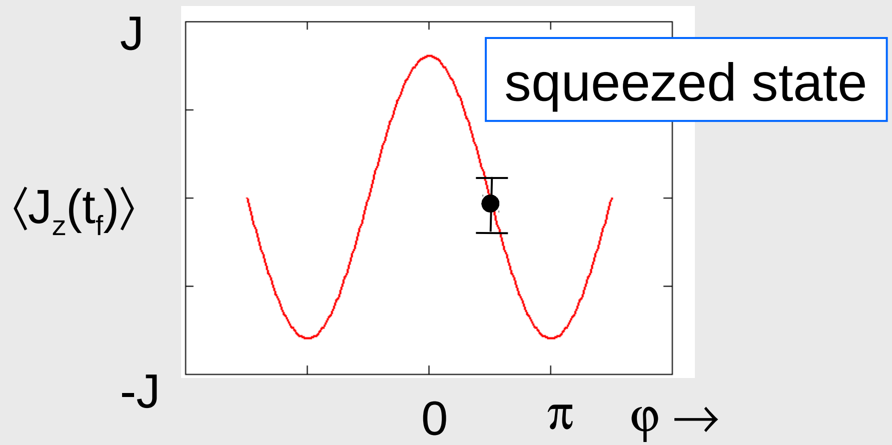




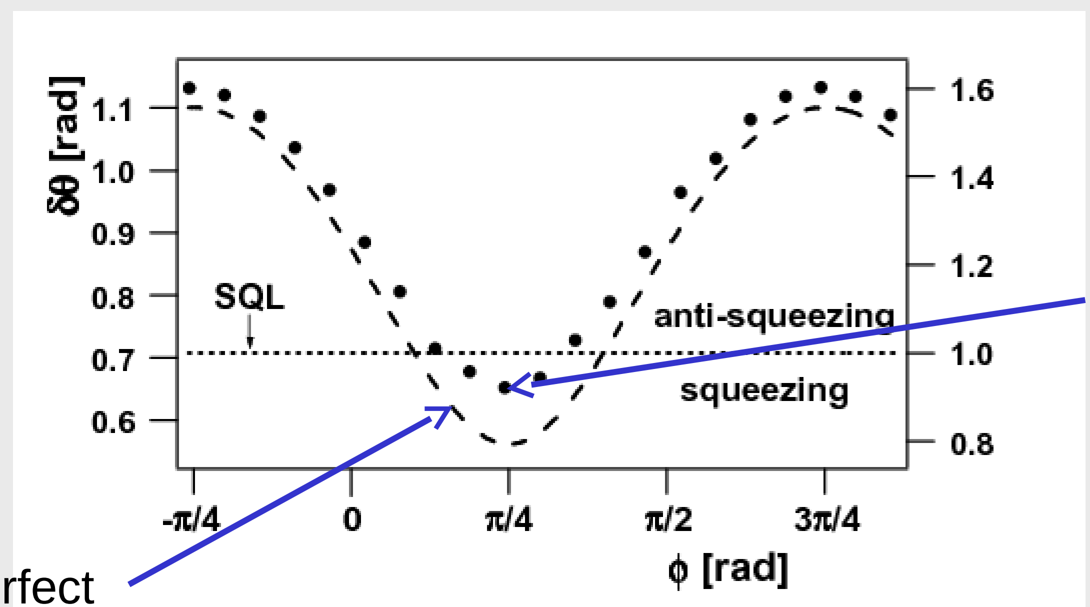
$$\kappa = \frac{|\langle \mathbf{J} \rangle|}{\Delta \mathbf{J}_\perp \sqrt{2J}}$$

V. Meyer *et al.*, PRL **86**, 5870 (2001)





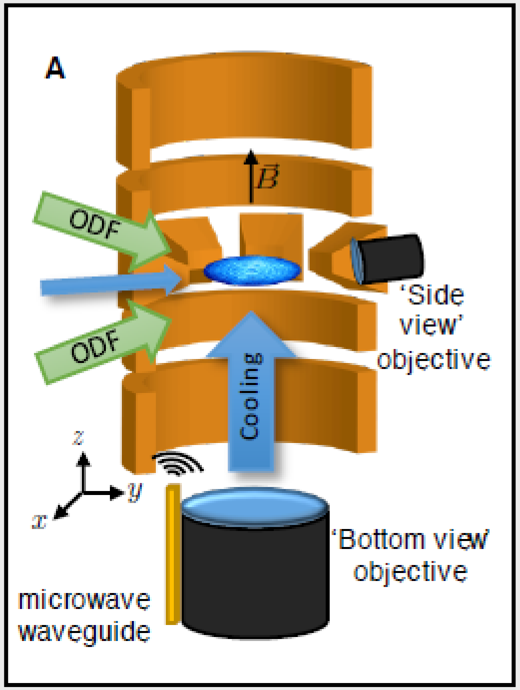
$$\Psi = \cos(\pi/10)|\downarrow\downarrow\rangle + \sin(\pi/10)|\uparrow\uparrow\rangle$$



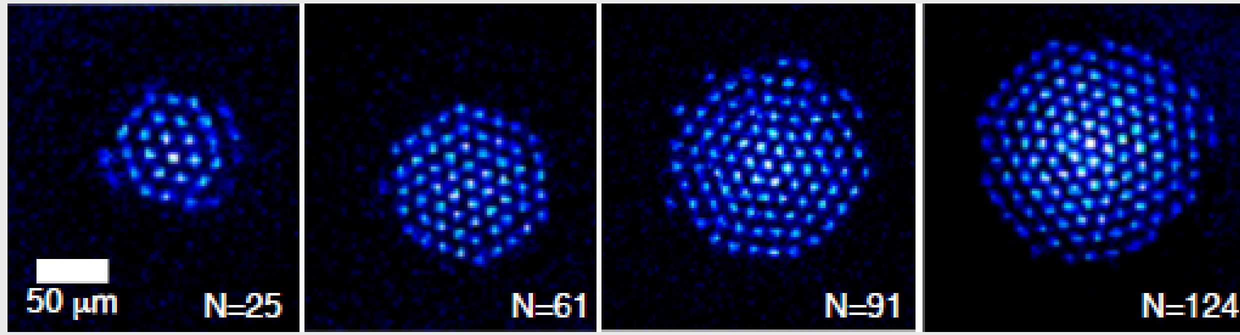
$\kappa \cong 1.07$, $N = 2$

for Ψ perfect

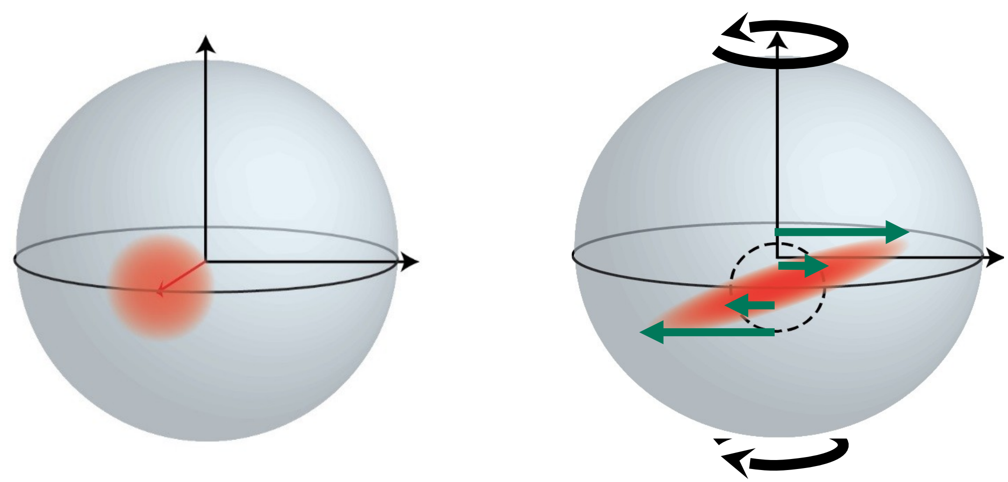
V. Meyer *et al.*, PRL **86**, 5870 (2001)



J. G. Bohnet, B. C. Sawyer, J. W. Britton, M. L. Wall, A. M. Rey, M. Foss-Feig, J. J. Bollinger (NIST)
Science **352**, 1297 (2016)



$$H_I = (-\chi J_z)\, J_z$$

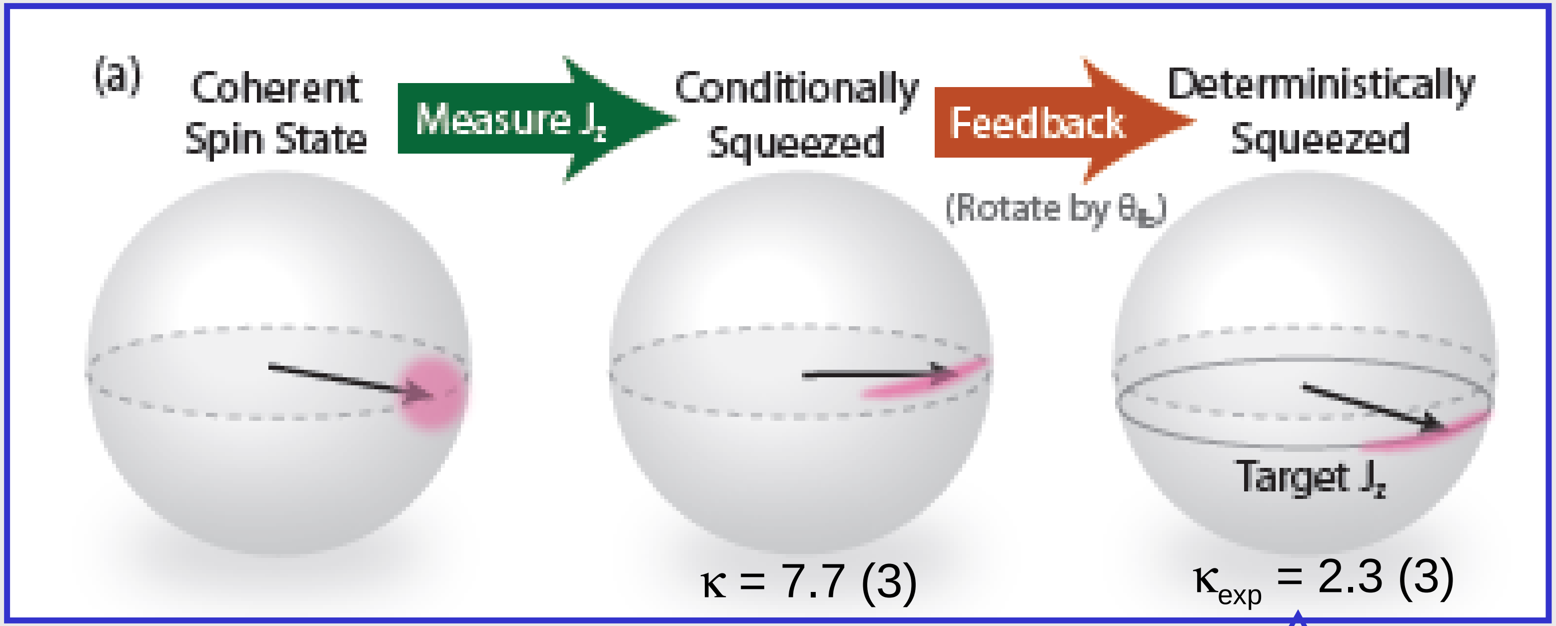


$$\kappa = 2.5\ (2), \quad N = 85$$

K. C. Cox, G. P. Greve, J. M. Weiner, and J. K. Thompson, PRL **116**, 093602 (2016)

^{87}Rb, $|\downarrow\rangle = |F=2, m_F = 2\rangle$, $|\uparrow\rangle = |3,3\rangle$

measure J_z (via cavity frequency pulling)
project to squeezed state



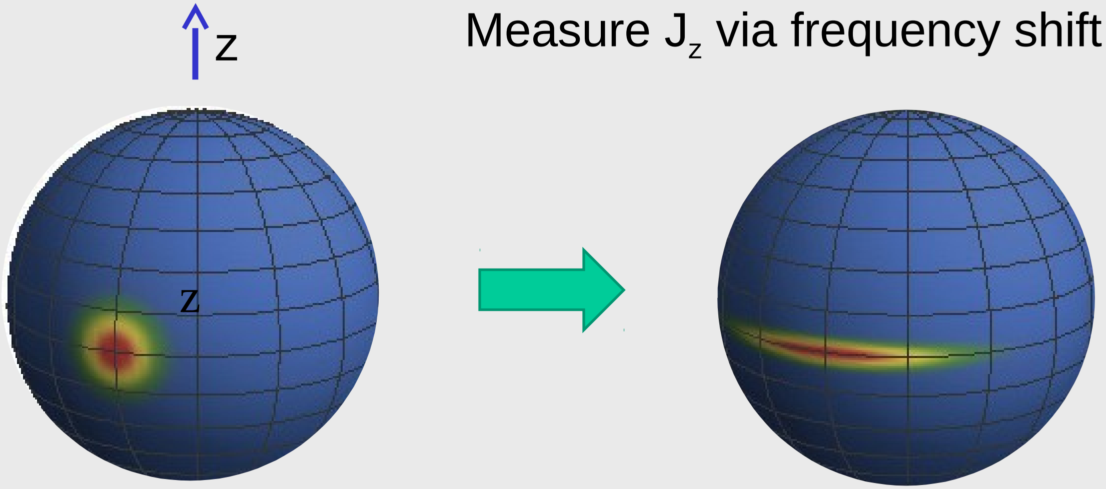
N = 4 x 10^5

squeezing reduced
by noise in feedback

O. Hosten, N. J. Engelsen, R. Krishnakumar, M. A. Kasevich
Nature **529**, 505 (2016)

^{87}Rb, $|\downarrow\rangle = |F=2, m_F = 0\rangle$, $|\uparrow\rangle = |3,0\rangle$

Measure J_z via frequency shift of coupled cavity

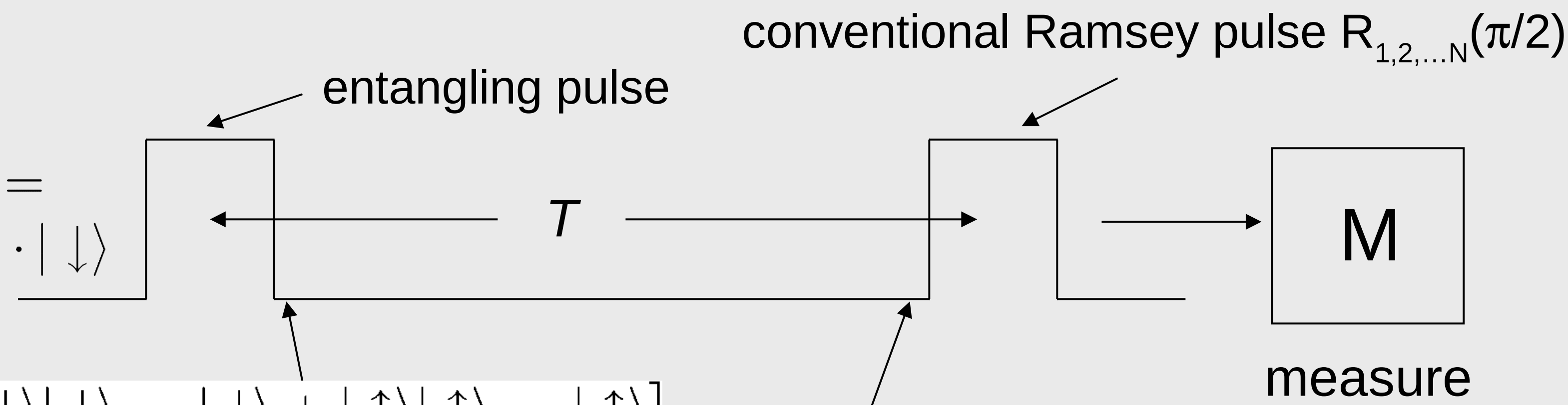


N = 5 x 10^5

κ = 10.1(2)

Ramsey interferometry of ⟨**J**⟩ = 0 states:

$$\Psi(t=0) = |\downarrow\rangle|\downarrow\rangle\cdots|\downarrow\rangle$$



$$\frac{1}{\sqrt{2}}\left[|\downarrow\rangle|\downarrow\rangle\cdots|\downarrow\rangle + |\uparrow\rangle|\uparrow\rangle\cdots|\uparrow\rangle\right]$$

$$\frac{1}{\sqrt{2}}\left[|\downarrow\rangle|\downarrow\rangle\cdots|\downarrow\rangle + e^{-iN\omega_0 T}|\uparrow\rangle|\uparrow\rangle\cdots|\uparrow\rangle\right]$$

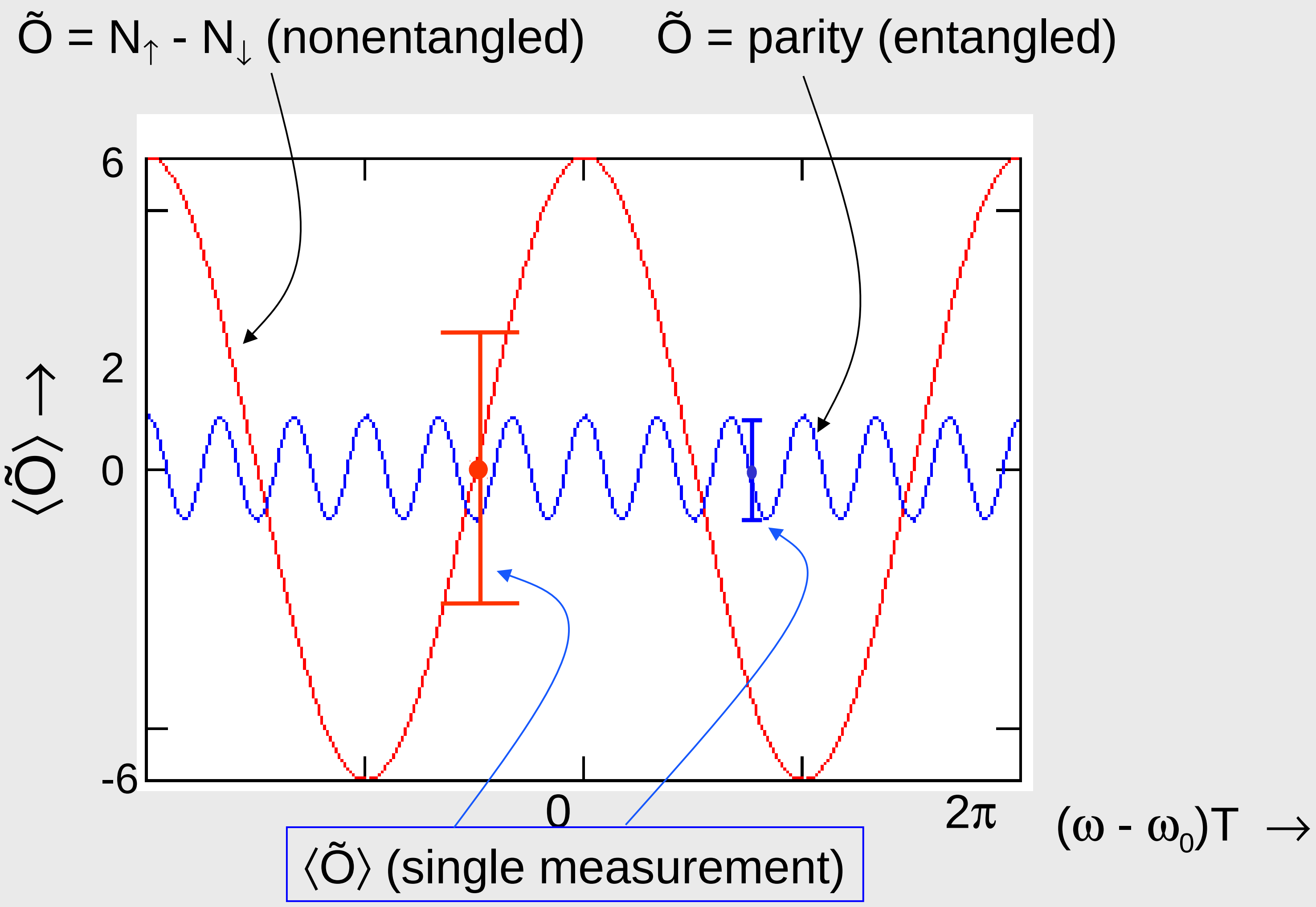
|⟨**J**⟩| = 0 !

spin "Schrödinger cat" states
⇒ Heisenberg limit

After second Ramsey pulse, measure parity operator:
(J. Bollinger *et al*., Phys. Rev. A**54**, R4649 (1996))

$$\tilde{O} = \tilde{P} = \prod_{i=1}^{N} \sigma_{zi}$$ possible: 1, -1)

e.g., N = 6



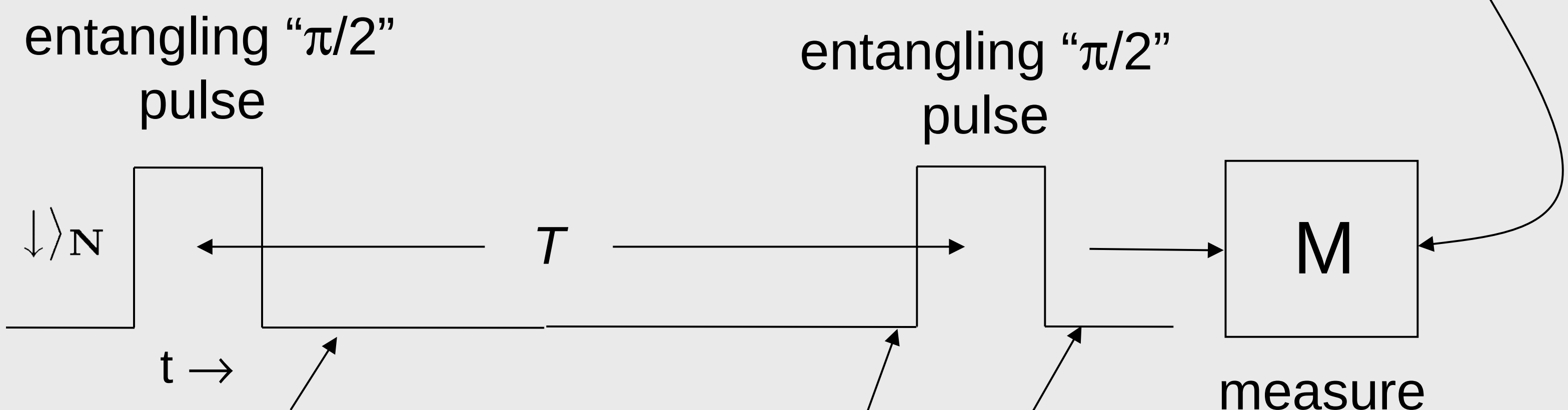
$\Delta\varphi = 1/N$, independent of $(\omega - \omega_0)T$ "Heisenberg limit"

Ramsey interferometry with two entangling pulses

(D. Leibfried *et al.* *Science* ’04, *Nature* ‘05)

$$\mathbf{P}_{\downarrow\downarrow\cdots\downarrow} = \frac{\mathbf{1}}{\mathbf{2}}\left(\mathbf{1} - \cos \mathbf{N}(\omega - \omega_{\mathbf{0}})\mathbf{T}\right)$$

$$|\downarrow\rangle_{\mathbf{1}}|\downarrow\rangle_{\mathbf{2}}\cdots|\downarrow\rangle_{\mathbf{N}}$$



$$\frac{1}{\sqrt{2}}\left[|\downarrow\rangle|\downarrow\rangle\cdots|\downarrow\rangle + |\uparrow\rangle|\uparrow\rangle\cdots|\uparrow\rangle\right]$$

$$\frac{1}{\sqrt{2}}\left[|\downarrow\rangle_1|\downarrow\rangle_2\cdots|\downarrow\rangle_N + e^{-iN\omega_0 T}|\uparrow\rangle_1|\uparrow\rangle_2\cdots|\uparrow\rangle_N\right]$$

$$\sin\big(N(\omega-\omega_0)T/2\big)\underbrace{|\downarrow\rangle_1\cdots|\downarrow\rangle_N}_{\text{all ions fluoresce}} + \cos\big(N(\omega-\omega_0)T/2\big)\underbrace{|\uparrow\rangle_1\cdots|\uparrow\rangle_N}_{\text{no ions fluoresce}}$$

Entangled state interferometry:

"Signal"

$$\partial\langle\tilde{P}_{\downarrow\downarrow\cdots\downarrow}(t_f)\rangle/\partial\phi = \frac{C}{2}\sin\phi$$

Noise

$$\Delta\tilde{P}_{\downarrow\downarrow\cdots\downarrow}(t_f) = \frac{1}{2}\sin\phi$$

S/N = C
"win" if C > $1/\sqrt{N}$

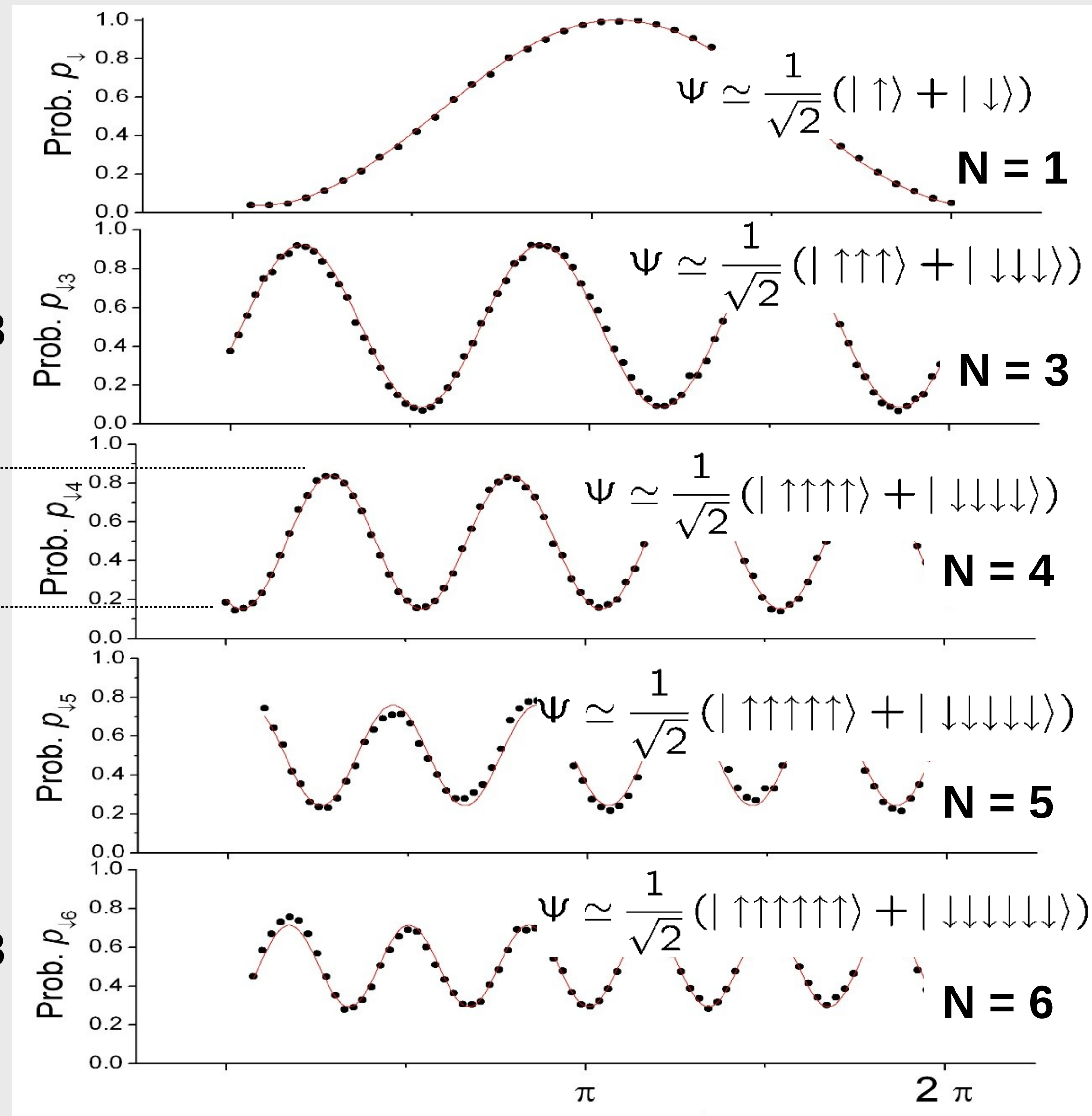
C = 0.84(1)
> $3^{-1/2}$ = 0.58
κ = 1.45(2)

C

T. Monz et al. (Innsbruck)
PRL 106, 130506 (2011)
κ > 1 for N = 14

C = 0.419(4)
> $6^{-1/2}$ = 0..408
κ = 1.03(1)

(D. Leibfried *et al. Science* 2004, *Nature* 2005)



Entangled state interferometry:

But! assumptions above:

perfec

Restr

• Huelg

PRL **79**

(phase

Future:

- More and better
- application of entangled states
  - to accurate clocks
  - to metrology
  - ??
- ….

• DJW

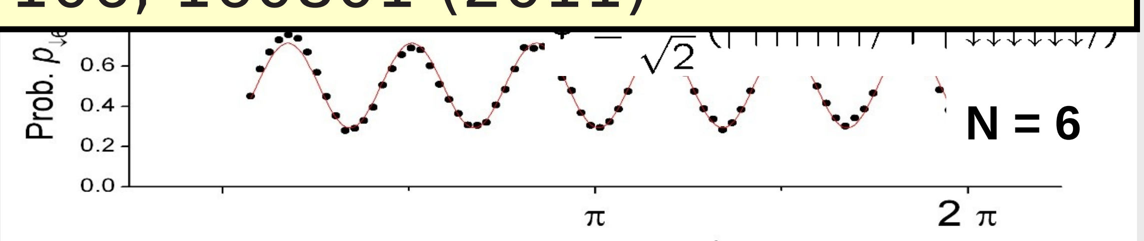
• André

(phase

• C. W. Chou et al., PRL 106, 160801 (2011)

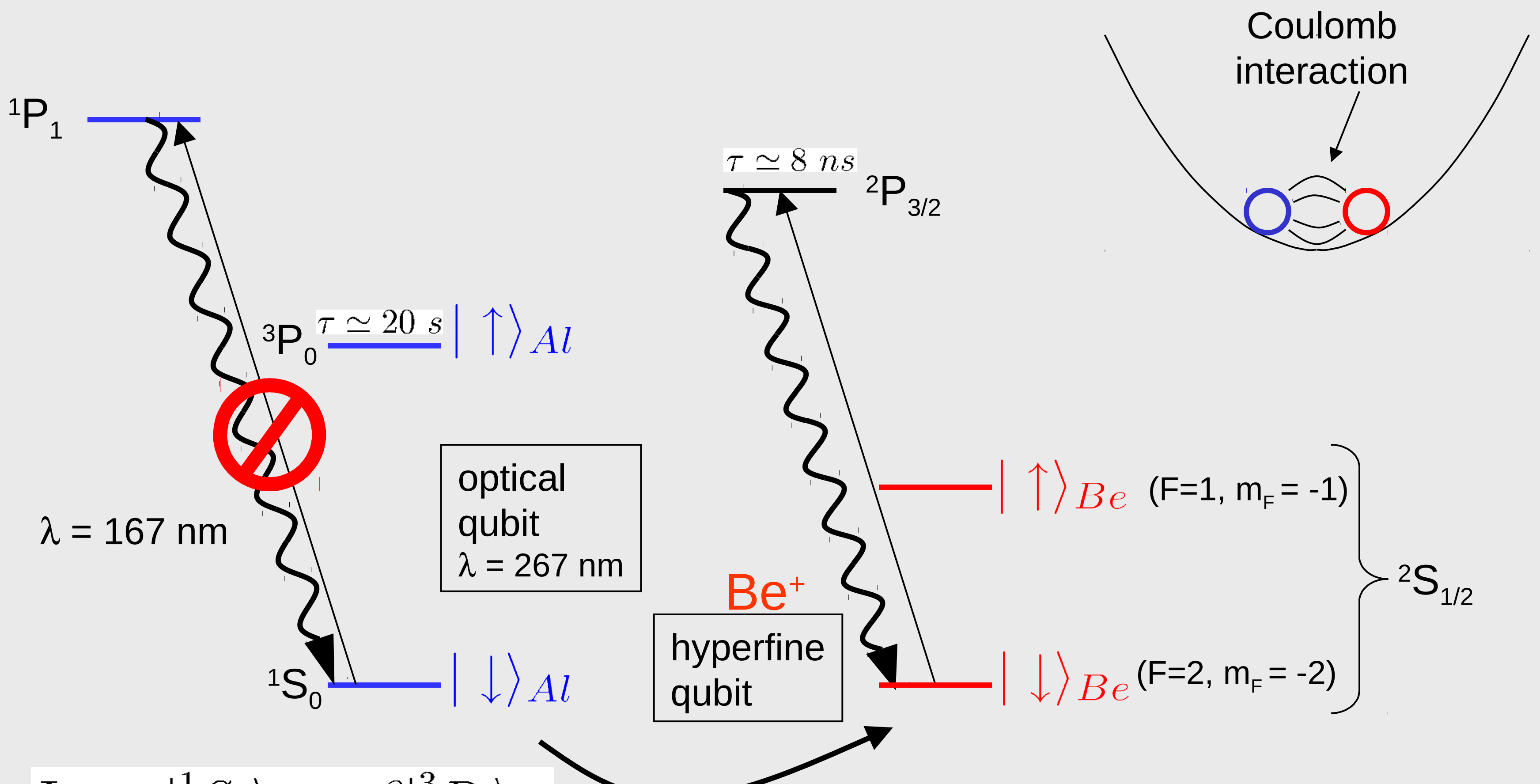
$> 6^{-1/2} = 0..408$

$\kappa = 1.03(1)$



Efficient detection with ancilla qubits

(Al+ optical clock experiment, NIST, Boulder)



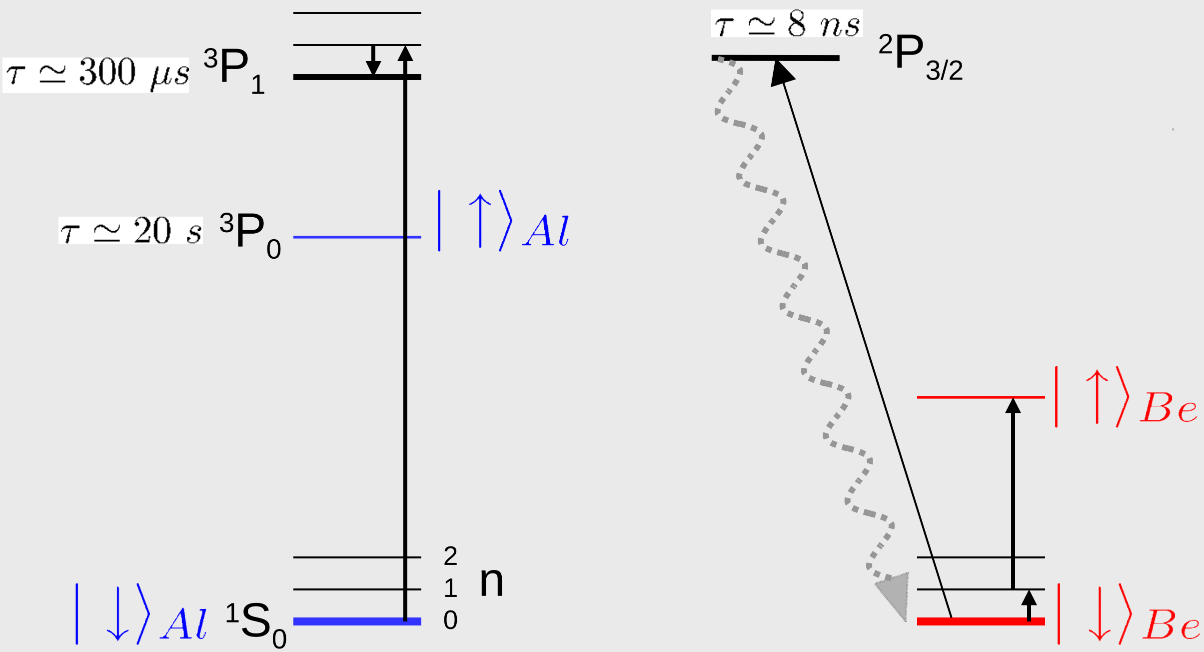
$$\Psi = \alpha|^1S_0\rangle_{Al} + \beta|^3P_0\rangle_{Al}$$

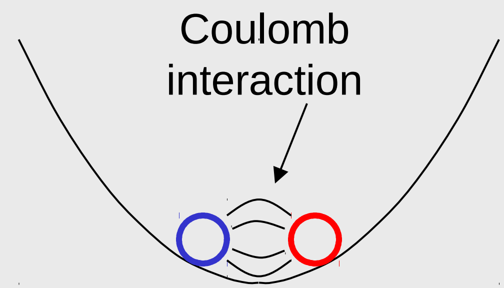
transfer information to $^9Be^+$

P. O. Schmidt et al., *Science* **309**, 749 (2005)

Efficient detection with ancilla qubits

(Al^+ optical clock experiment, NIST, Boulder)





Cool ions to ground state with Be^+: $\Psi = |^1S_0\rangle_{Al}$ $|\downarrow\rangle_{Be}$ $|n=0\rangle$

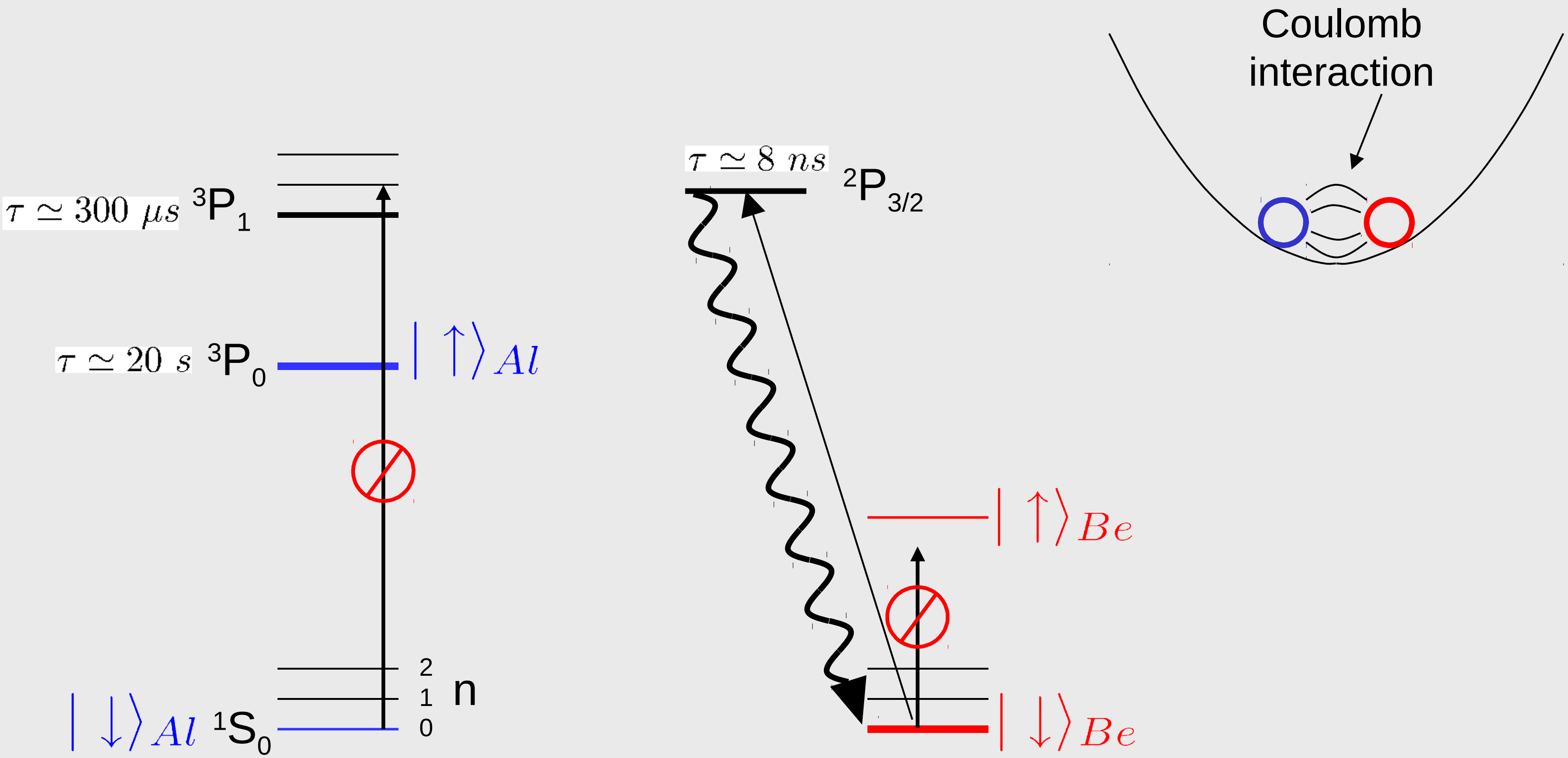
$$|^1S_0\rangle_{Al}|\downarrow\rangle_{Be}|0\rangle \rightarrow \text{blue sb}(\pi)_{Al} \rightarrow |^3P_1\rangle_{Al}|\downarrow\rangle_{Be}|1\rangle$$

$$\rightarrow \text{red sb}(\pi)_{Be} \rightarrow |^3P_1\rangle_{Al}|\uparrow\rangle_{Be}|0\rangle$$

P. O. Schmidt et al., *Science* **309**, 749 (2005)

Efficient detection with ancilla qubits

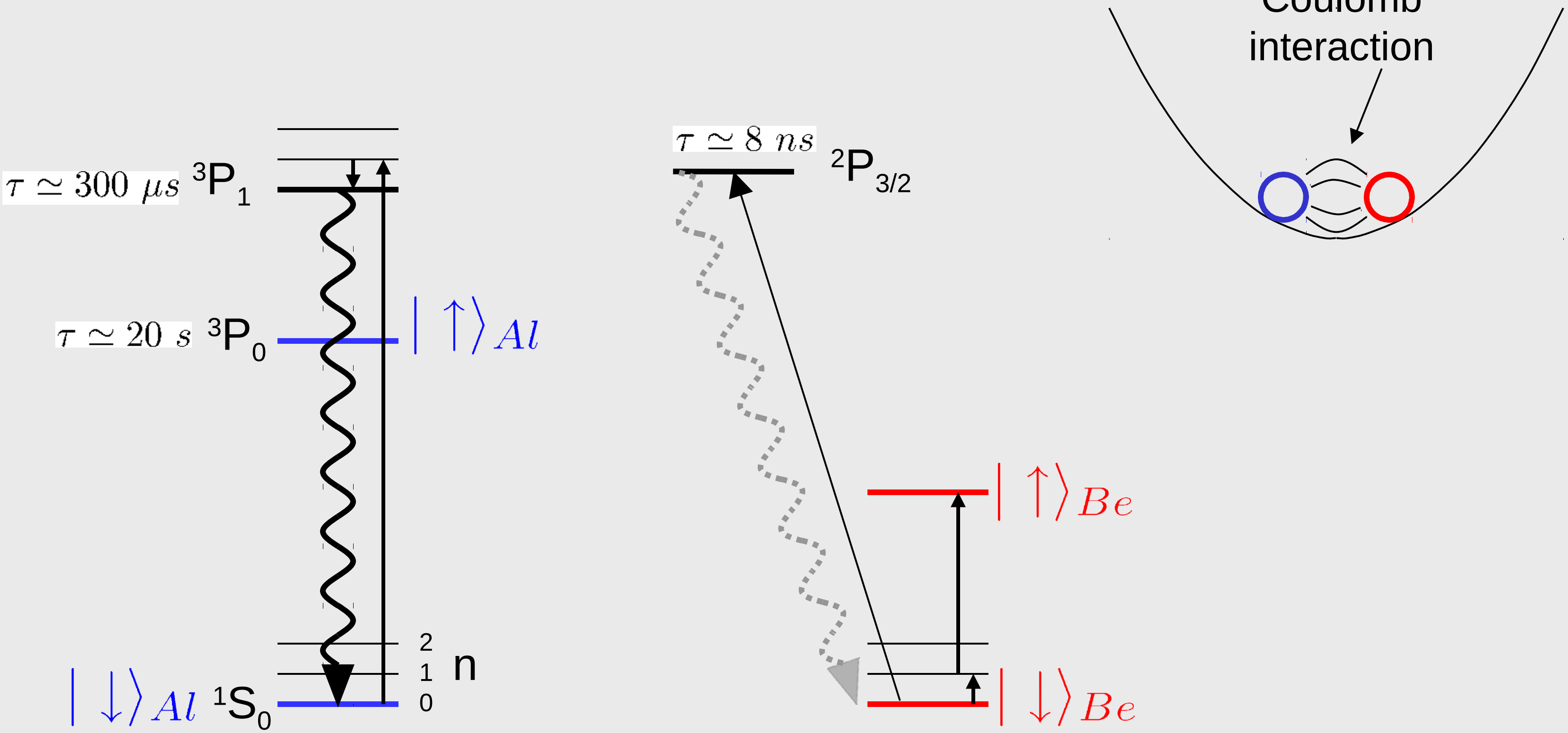
(Al$^+$ optical clock experiment, NIST, Boulder)



If $\Psi = |^3P_0\rangle|\downarrow\rangle_{Be}|n=0\rangle$

P. O. Schmidt et al., *Science* **309**, 749 (2005)

Efficient detection with ancilla qubits
(Al⁺ optical clock experiment, NIST, Boulder)

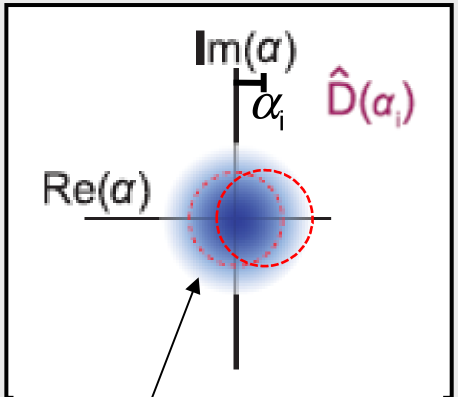


Is "QND" measurement; can repeat to increase Fidelity
F = 0.85 → 0.9994
D. Hume *et al.*, Phys. Rev. Lett. **99**, 120502 (2007)

Now, extensions to molecules: C.–W. Chou et al., Nature 545, 203 (2017).

P. O. Schmidt et al., *Science* **309**, 749 (2005)

Sensitive mechanical motion detection
e.g. single harmonic motion

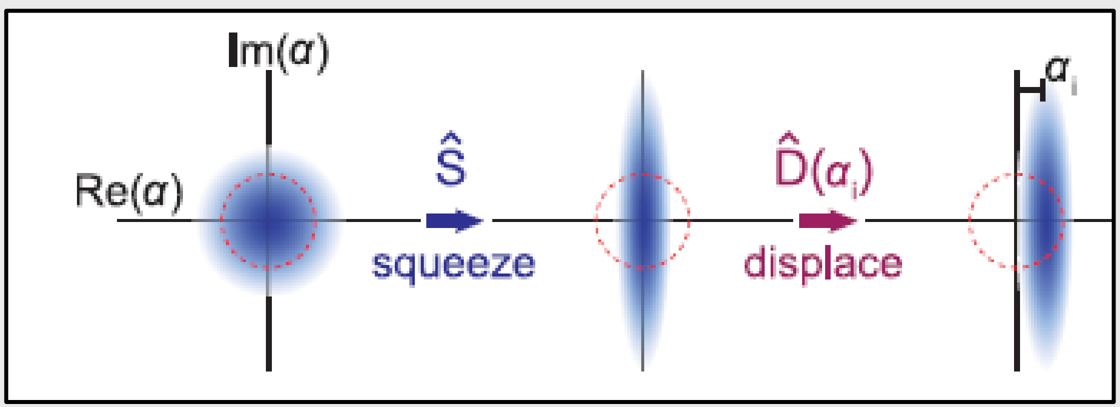


ground state (n =0) wavefunction

$\hat{D}(\alpha_i) \equiv \exp[\alpha_i a^{\dagger} - \alpha_i^* a]$ with $\alpha = (\Delta x + i\Delta p/m\omega_x)/(2x_0))$

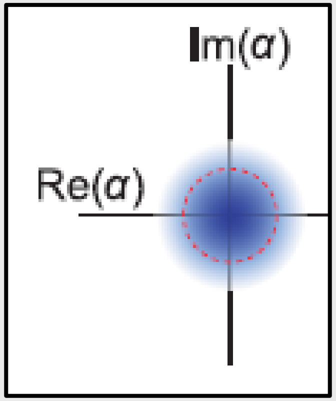
here, $\kappa \propto \alpha_I$ / ground-state wave packet spread

$$\hat{S}(\xi) = \exp[(\xi^* \hat{a}^2 - \xi \hat{a}^{\dagger 2})/2]$$

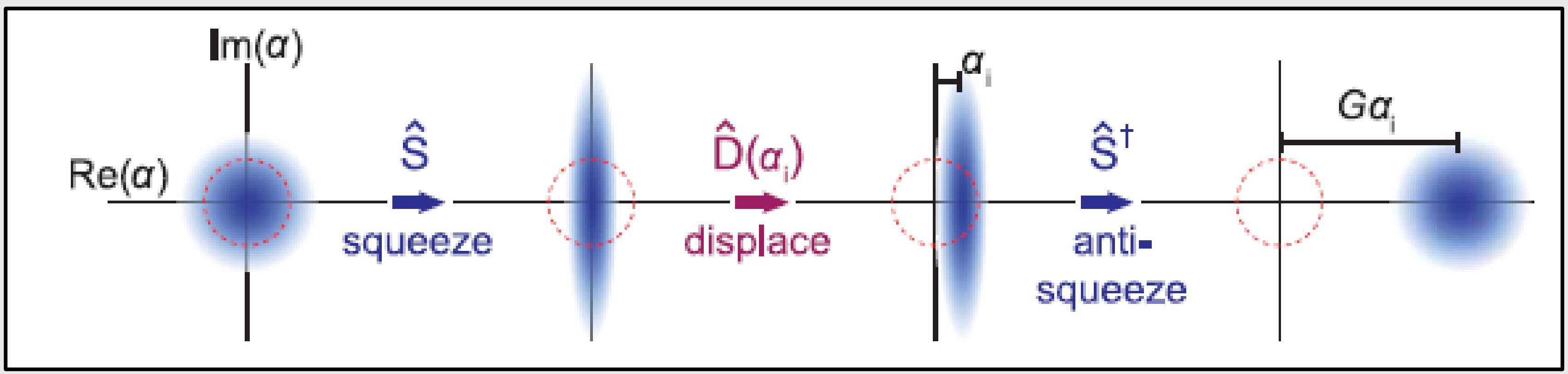


$$\hat{D}(\alpha_f) = \hat{S}^{\dagger}(\xi)\hat{D}(\alpha_i).$$

Sensitive mechanical motion detection
e.g. single harmonic motion

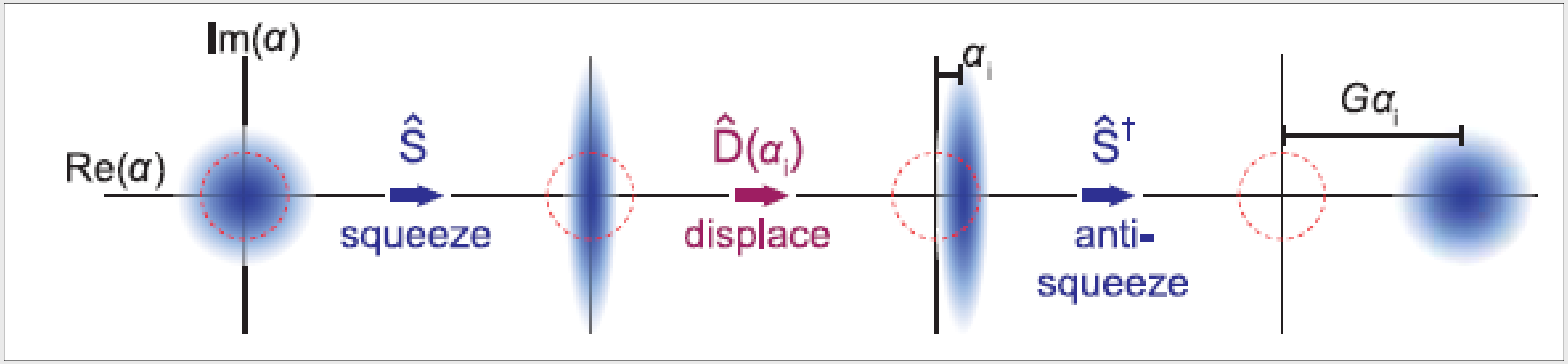


$(D(\alpha) \equiv \exp[\alpha a^{\dagger} - \alpha^{*} a]$ with $\alpha = (\Delta x + i\Delta p/m\omega_x)/(2x_0))$



$$\hat{D}(\alpha_f) = \hat{S}^{\dagger}(\xi)\hat{D}(\alpha_i)\hat{S}(\xi)$$

$$\hat{D}(\alpha_f) = \hat{S}^\dagger(\xi)\hat{D}(\alpha_i)\hat{S}(\xi)$$



$\hat{S}(\xi) = \exp[(\xi^* \hat{a}^2 - \xi \hat{a}^{\dagger 2})/2]$ generated with parametric drive

trap potential modulated at $2\omega_x$

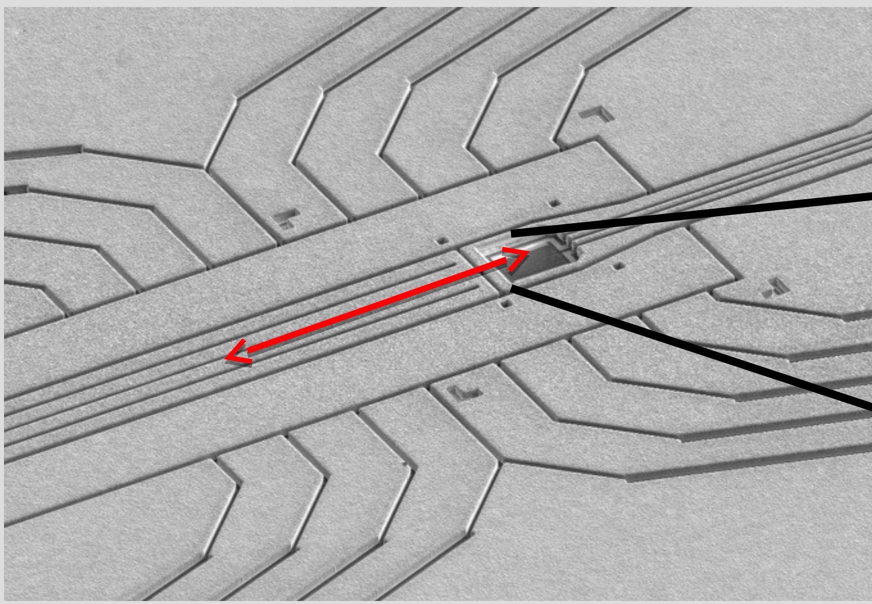
$\kappa \cong 7.3$

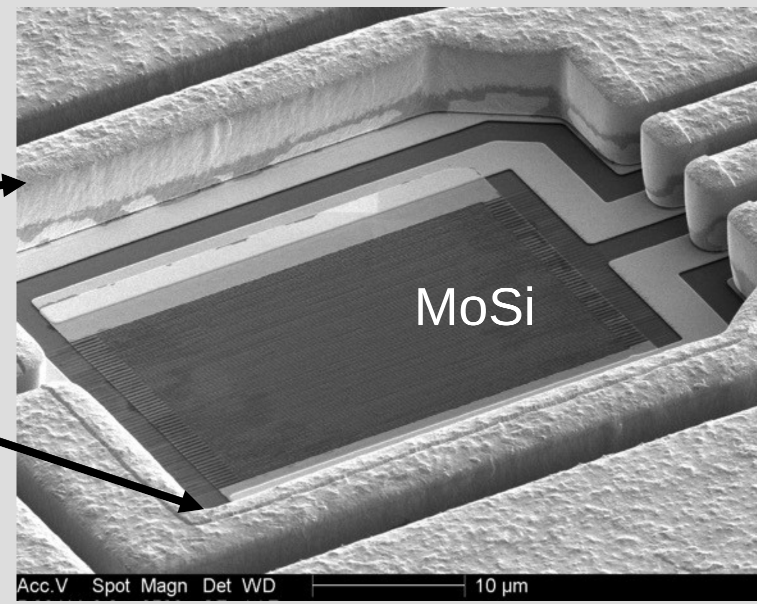
S. C. Burd et al., Science **364**, 1163 (2019).

Photon metrology:

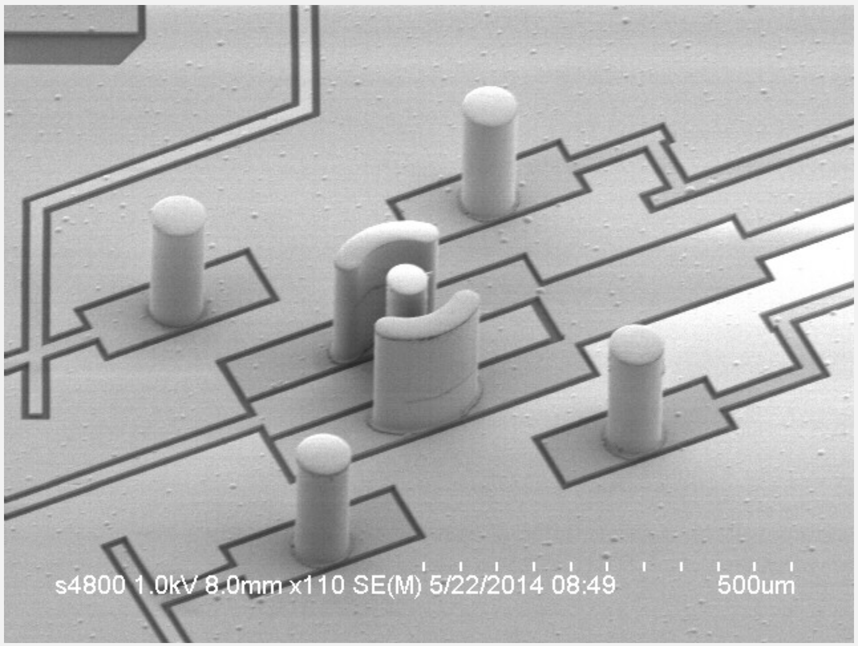
- UV fibers: Y. Colombe et al., Optics Express, **22**, 19783 (2014)
recipe: http://www.nist.gov/pml/div688/grp10/index.cfm

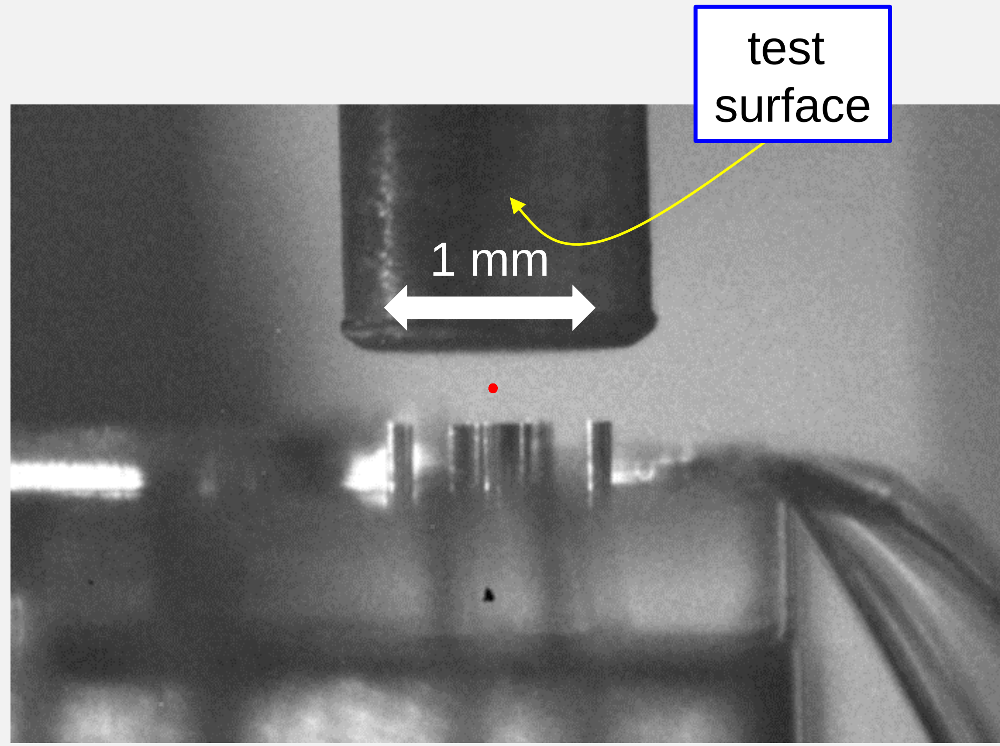
- Detection without optics; D. Slichter, V. Verma





Surface metrology:
"Stylus" trap for ion-heating studies
D. Hite, K. McKay, et al., NIST



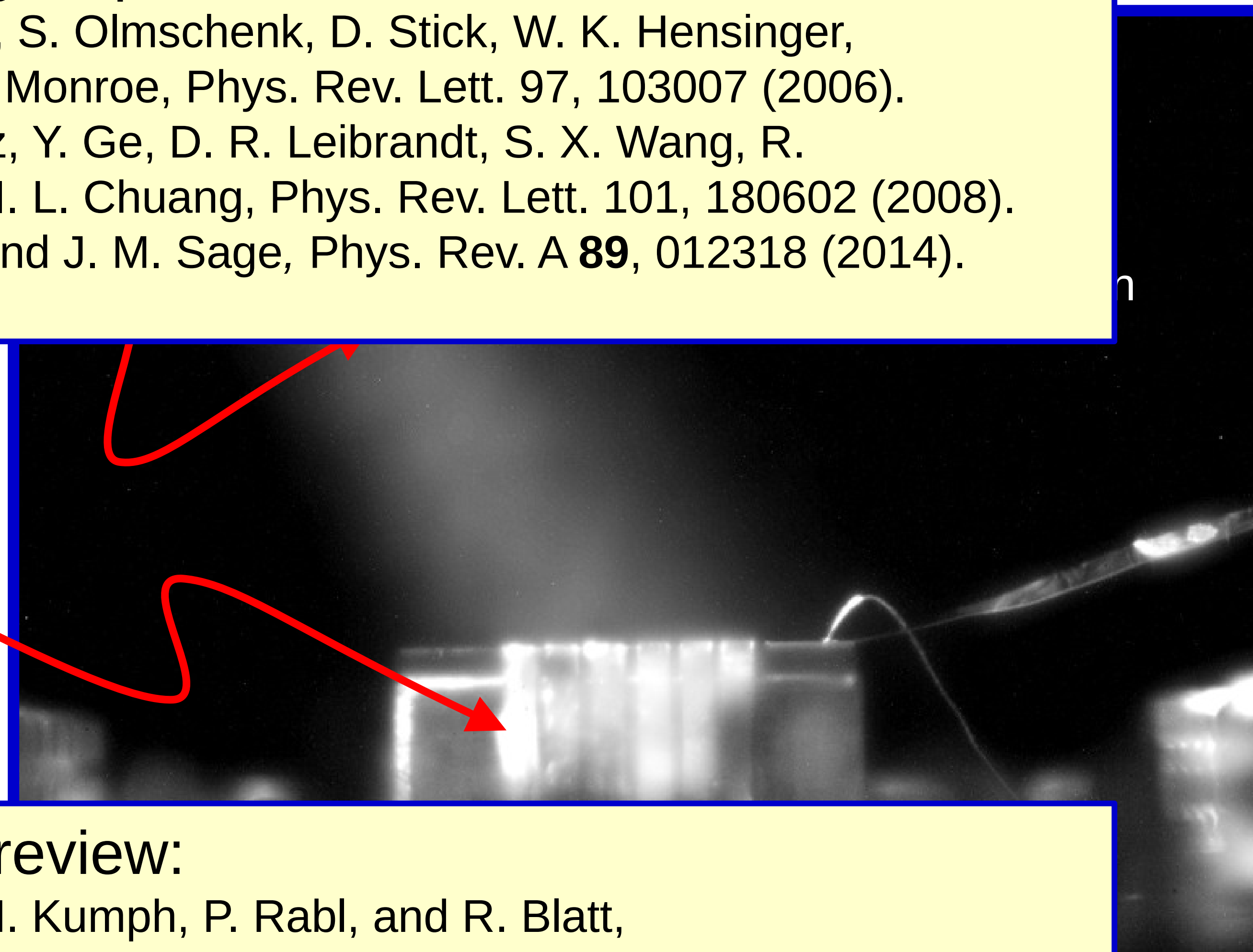


D. A. Hite, et al., *MRS Advances*, pp. 1–9. doi: 10.1557/adv.2017.14. (2017).

Ion heating: try to reduce with surface science techniques:

~ 2 kV
p(Ar) ~

cleaning



side view,
surface-electrode trap

Johnson

Cryo cooling helps too:

- L. Deslauriers, S. Olmschenk, D. Stick, W. K. Hensinger, J. Sterk, and C. Monroe, Phys. Rev. Lett. 97, 103007 (2006).
- J. Labaziewicz, Y. Ge, D. R. Leibrandt, S. X. Wang, R. Shewmon, and I. L. Chuang, Phys. Rev. Lett. 101, 180602 (2008).
- J. Chiaverini and J. M. Sage, Phys. Rev. A **89**, 012318 (2014).

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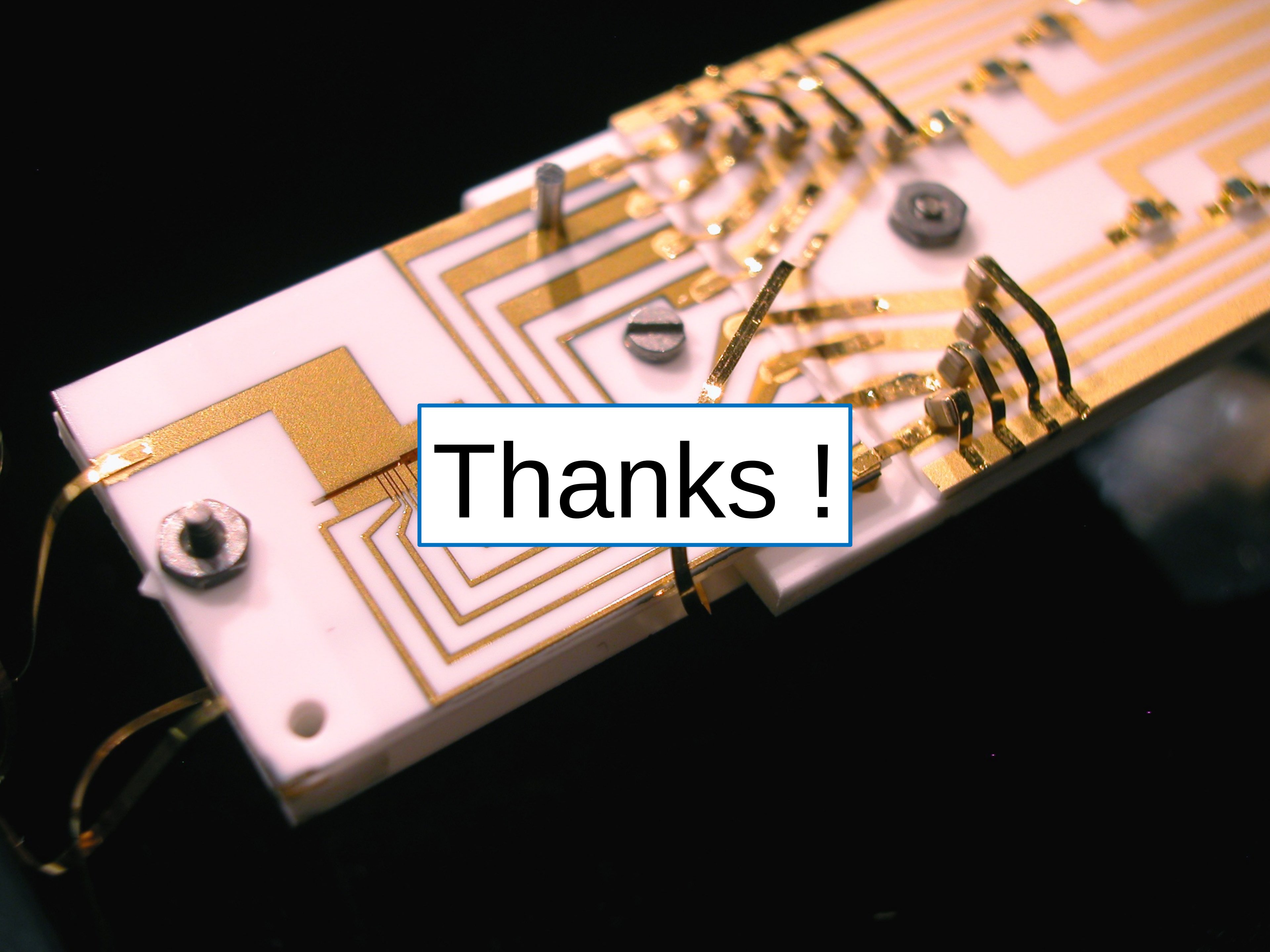
Ion heating review:

M. Brownnutt, M. Kumph, P. Rabl, and R. Blatt,
RMP **87**, 1419 (2015)

D. A. Hite et al., PRL **109**, 103001 (2012) (Ar^+ beam sputtering)

N. Daniilidis et al., (Häffner group) PRB **89**, 245435 (2014): similar gain



Thanks !